FVRL: Fleming-Viot Reinforcement Learning for the efficient exploration of environments with sparse and rare rewards

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Relevant keywords and concepts

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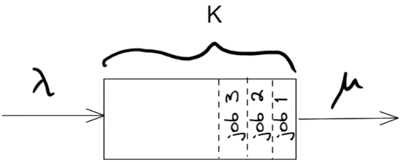
- Reinforcement Learning (tackles Markov Decision Processes)
  - Exploration (impacts Learning)
  - Rewards (sparse and rare) (impacts Learning)

Motivation - The M/M/1/K queue system



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$$\lambda < \mu$$

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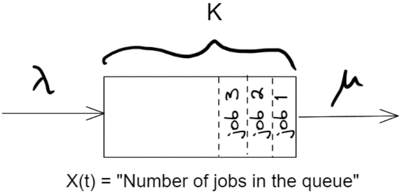

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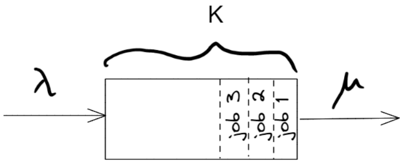

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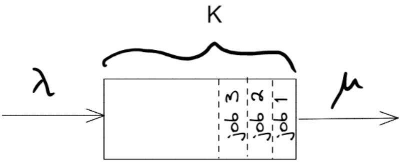

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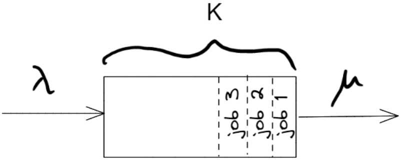

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Although rare, blocking can be very costly when it happens...

Outline

Environments with **sparse and rare rewards**

Fleming-Viot particle systems for **probability estimation**

FVRL: Fleming-Viot particle systems for **learning optimum policy**

Many environments give very few rewards - sparsity

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- Commonly used to estimate prob. rare events:
  Importance Sampling
  $\rightarrow$ *Here we explore a completely different approach, using Fleming-Viot particle systems*

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E.g. location of zero-reward states

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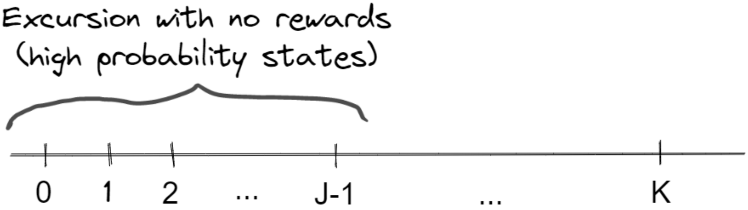
  M/M/1/K queue: set of queue sizes where no blocking has happened



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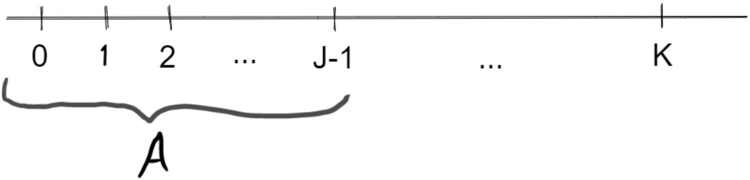

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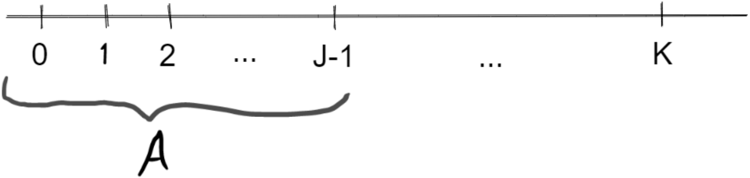

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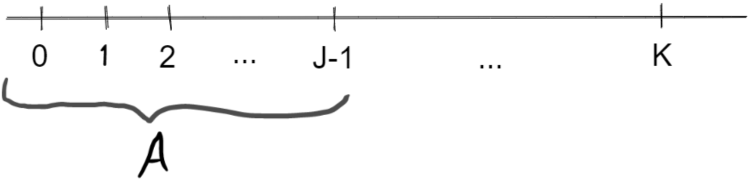

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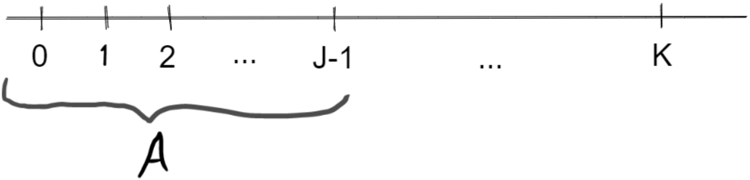

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- When **absorbed** $\rightarrow$ **reactivation** to one of other $N-1$ particles

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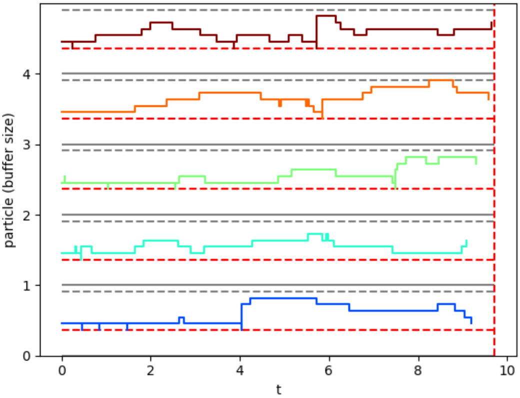
- FV dynamics example on $N = 5$ particles, $K = 10$, $J = 5$



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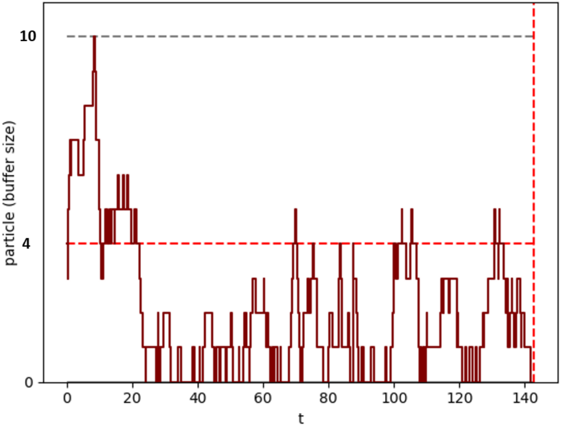
- MC dynamics (same number of events as FV case)



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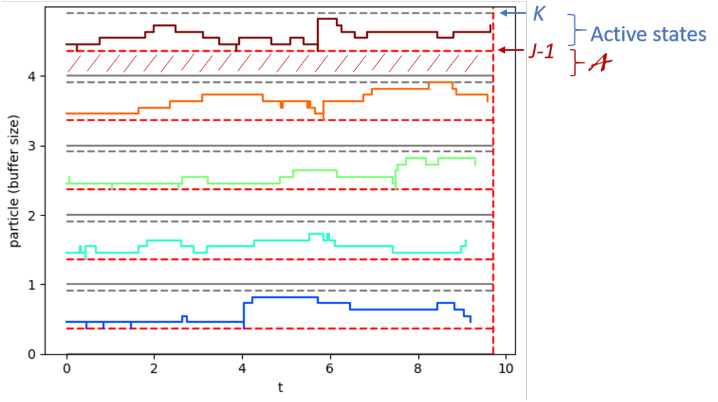

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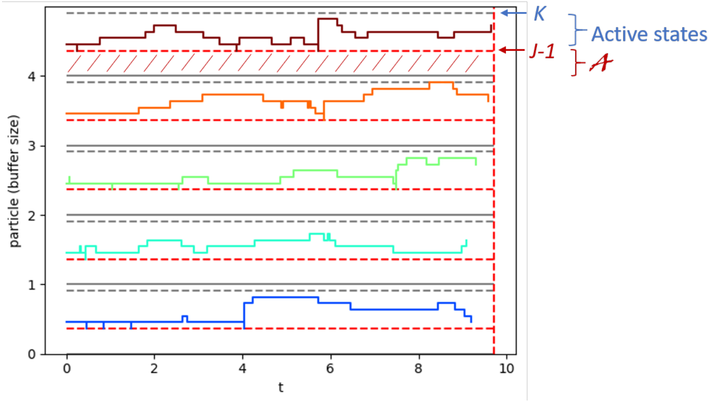


First we need to know how to estimate probabilities using FV

Stationary probability estimation using FV

Assume **irreducible, aperiodic** Markov chain X_t, **renewal theory** gives us a characterization of the **stationary probability** of state x:

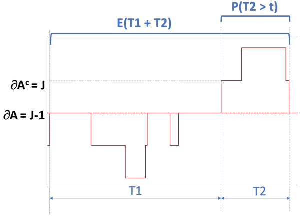
$$p(x) = \frac{\mathbb{E}^i\left(\int_0^{T_i} \mathbb{1}\{X_t = x\}dt\right)}{\mathbb{E}^i T_i} = \frac{\int_0^{\infty} \mathbb{E}^i \mathbb{1}\{X_t = x, t \le T_i\}dt}{\mathbb{E}^i T_i}$$

T_i is the random cycle time: state $i \to i$ (*any* chosen state i).

Stationary probability estimation using FV

Given knowledge of set $\mathcal{A}$ of zero-reward states, it's convenient to write, for $x \notin \mathcal{A}$:

$$p(x) = \frac{\int_0^\infty \phi_t^{\partial \mathcal{A}^c}(x) P^{\partial \mathcal{A}^c}(T_2 > t) dt}{\mathbb{E}^{\partial \mathcal{A}}\left[T_1 + T_2\right]}$$



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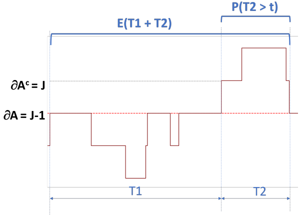

$\partial\mathcal{A}^c$ is the boundary of the complement set of $\mathcal{A}$,
$\phi_t^{\partial\mathcal{A}^c}(x) = P^{\partial\mathcal{A}^c}(X_t = x | T > t)$
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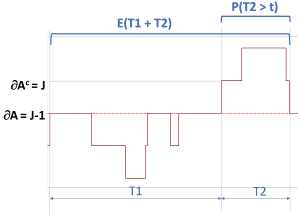

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Key fact ([Asselah et al.(2011)Asselah, Ferrari, and Groisman]):
"Empirical probability of x" $\rightarrow \phi_t^{\partial\mathcal{A}^c}(x)$ in L_1-norm when $N \rightarrow \infty$ as $\frac{1}{\sqrt{N}}$

Estimation of $p(x), x \notin \mathcal{A}$

Illustration on M/M/1/K queue system:

$$p(x) = \frac{\int_0^\infty \phi_t^J(x) P^J(T_2 > t) dt}{\mathbb{E}^{J-1}\left[T_1 + T_2\right]}$$

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- Two simulations, *in order*:
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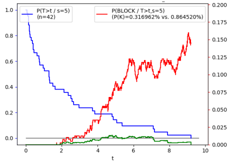
Flavour of $\hat{P}^J(T_2 > t)$, $\hat{\phi}_t^J(x)$, $\hat{\phi}_t^J(x)\hat{P}^J(T_2 > t)$ as function of t



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- **Smaller J:** Need larger number of particles N for proper estimation of numerator, $\phi_t^J(x)$

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- Fair comparison with vanilla Monte-Carlo (MC)

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Medium-size capacity K - FV and MC give unbiased estimates

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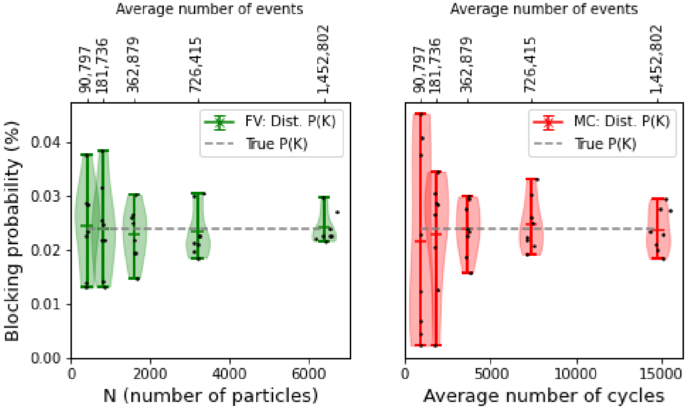
Convergence with number of particles N - FV vs. MC

$\mathbf{K = 20, Pr(K) \sim 10^{-4}}$, $J = K/2$, on 8 replications per violin plot



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Large capacity K - FV gives unbiased estimates while MC fails

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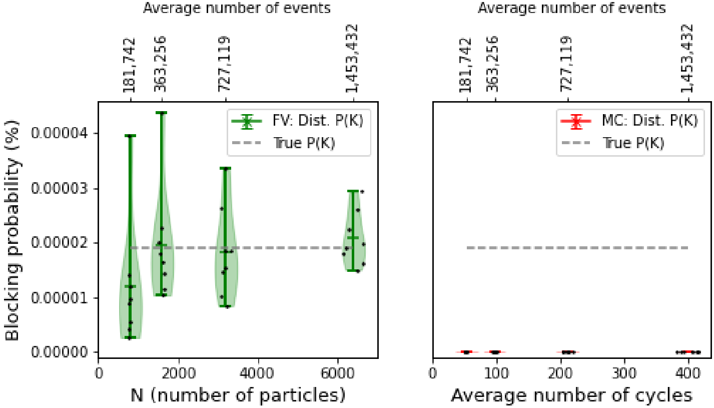

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  - **States** $x \in S$ and **Actions** $a \in A(x)$
  - **Rewards** for each state and action $(R(x, a))$
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  States, actions and rewards are observed in time: X_t, A_t, R_t

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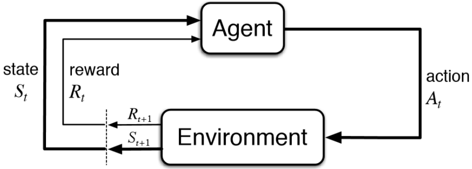
- however... system parameters are normally unknown
  $\implies$ **reinforcement learning** comes into rescue

Overview of reinforcement learning (RL)



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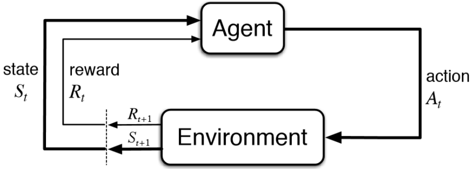


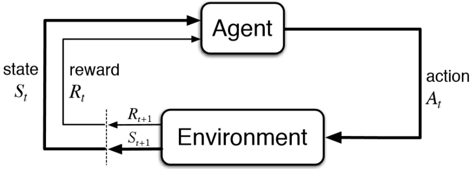
Given the rewards R_t and a **fixed policy** π, an underlying value for each state and action can be defined, as well as a value for each state:

$$Q^\pi(x,a) = \sum_{t=0}^{\infty} \mathbb{E}^\pi\left[(R_t - \overline{v}^\pi) \mid S_0 = x, A_0 = a\right]$$

$$v^\pi(x) = \sum_{t=0}^{\infty} \mathbb{E}^\pi\left[(R_t - \overline{v}^\pi) \mid S_0 = x\right]$$

where $\overline{v}^\pi := \sum_x p^\pi(x) \sum_a \pi(a|x) Q^\pi(x,a)$ (**average long-run reward**)

Overview of reinforcement learning (RL)



Given the rewards R_t and a **fixed policy** π, an underlying value for each state and action can be defined, as well as a value for each state:

$$Q^\pi(x,a) = \sum_{t=0}^{\infty} \mathbb{E}^\pi \left[(R_t - \overline{v}^\pi) \mid S_0 = x, A_0 = a \right]$$

$$v^\pi(x) = \sum_{t=0}^{\infty} \mathbb{E}^\pi \left[(R_t - \overline{v}^\pi) \mid S_0 = x \right]$$

where $\overline{v}^\pi := \sum_x p^\pi(x) \sum_a \pi(a|x) Q^\pi(x,a)$ (**average long-run reward**)

Goal: find policy that optimises the average long-run reward $\overline{v}^\pi$

Queue environment model

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Non-zero only when incoming job is **blocked**

B, b and x_{ref} are positive constants

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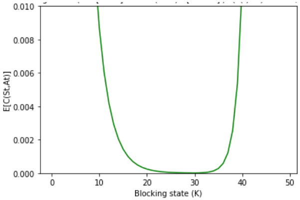
Example of **Expected** cost vs. K

$$\mathbb{E}^{\pi}\left[R(X_t, A_t)\right] = 5(1 + 3^{K-30})p^{\pi}(K)$$



Policy gradient proposed to learn optimum K

Policy gradient theorem makes life eas(*ier*)

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- Parameterised policy by θ: $\pi_\theta(a|x), \theta \in \mathbb{R}$
- Gradient descent to find minimum average cost, $\overline{v}_\theta^{\pi *}$
- Policy gradient theorem [Sutton et al.(2000)Sutton, McAllester, Singh, and Mansour]

$$\begin{aligned}\nabla_\theta \overline{v}_\theta^\pi &= \mathbb{E}^{\pi_\theta}[Q_\theta(X,a)\nabla_\theta \pi_\theta(a|X)] \\ &= \sum_{x \in \mathcal{S}} p^{\pi_\theta}(x) \sum_a Q_\theta(x,a)\nabla_\theta \pi_\theta(a|x)\end{aligned}$$

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Proposed parameterised policy is a linear step function

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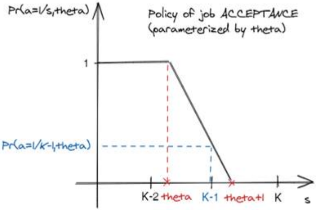
- Linear-step parameterised policy [Massaro et al.(2019)Massaro, Pellegrini, and Maggi]

Non-deterministic policy

$$\pi_\theta(a = 1|x) = \begin{cases} 1 & \text{if } x \leq \theta, \\ x - \theta + 1 & \text{if } \theta < x < \theta + 1 \\ 0 & \text{if } x \geq \theta + 1 \end{cases}$$



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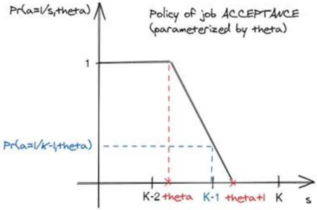

- Gradient becomes

$$\frac{\partial v_\theta^\pi}{\partial \theta} = p_\theta^\pi(K-1)\left[Q_\theta(K-1,1) - Q_\theta(K-1,0)\right]$$

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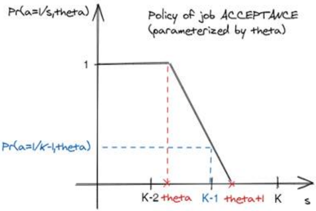

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$$\frac{\partial v_\theta^\pi}{\partial \theta} = \underbrace{p_\theta^\pi(K-1)}_{\text{Estimated with FV}} \underbrace{[Q_\theta(K-1,1) - Q_\theta(K-1,0)]}_{\text{Estimated with standard Monte-Carlo}}$$

FVRL results on the M/M/1/K queue system

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- Fair comparison with vanilla Monte-Carlo (MC)

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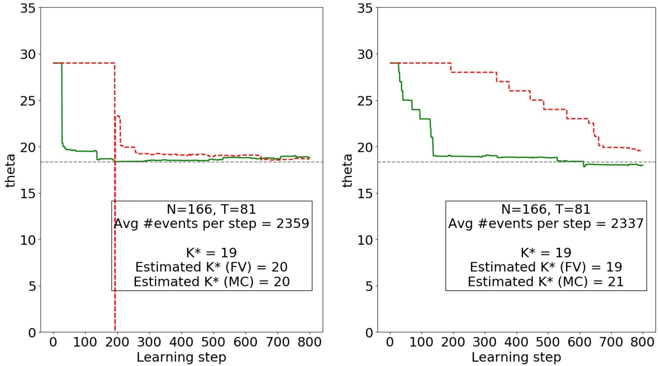
Moderate K^* - Fleming-Viot vs. Monte-Carlo

$$K^* = 19, K_0 = 30, J = 0.3K$$

"free" learning | clipped learning to ± 1



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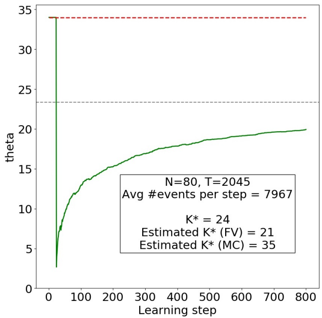
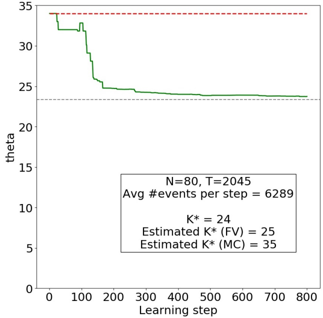
Large K^* - Fleming-Viot vs. Monte-Carlo

$$K^* = 24, K_0 = 35, J = 0.5K$$

"free" learning clipped learning to ± 1





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Thank you for your attention!

This presentation is based on the paper:

Mastropietro, D., Majewski S., Ayesta U., Jonckheere M.
"Boosting reinforcement learning with sparse and rare rewards using Fleming-Viot particle systems"
submitted to ICML 2022

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Sutton, R. S., McAllester, D. A., Singh, S. P., and Mansour, Y.
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