The multi-modal traveling salesman problem

Nicolas Jozefowiez[1], Gilbert Laporte[2], Frédéric Semet[3]

1. LAAS-CNRS, INSA, Université de Toulouse, Toulouse, France, nicolas.jozefowiez@laas.fr

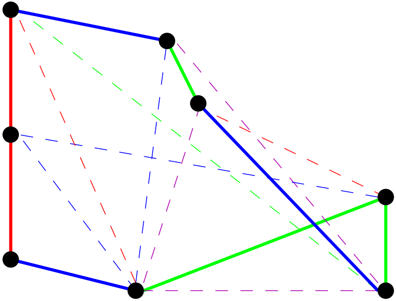
2. CIRRELT, HEC, Montréal, Canada, gilbert@crt.umontreal.ca

3. LAGIS, Ecole Centrale de Lille, Villeneuve d'Ascq, France, frederic.semet@ec-lille.fr

Outlines

- The multi-modal traveling salesman problem
- Multi-objective optimization
- A branch-and-cut algorithm
- Computational results
- Conclusions and perspectives

The multi-modal traveling salesman problem



Data:

$G = (V, E)$: an undirected valuated graph
C is a set of colors
Each $e \in E$ has a color $k \in C$

Goal:

Find a Hamiltonian cycle
Two objectives:

1. Minimize the total length of the cycle
2. Minimize the number of colors appearing on the cycle

Integer program

Variables

$$x_e = \begin{cases} 1 & \text{if } e \in E \text{ is used,} \\ 0 & \text{otherwise.} \end{cases}$$

$$u_k = \begin{cases} 1 & \text{if } k \in C \text{ is used,} \\ 0 & \text{otherwise.} \end{cases}$$

Constants and notations

$$\forall e \in E,\ \delta(e) = k \in C \text{ the color of } e$$

$$\forall k \in C,\ \zeta(k) = \{e \in E | \delta(e) = k\}$$

$$\forall S \subset V,\ \omega(S) = \{e = (i,j) \in E | i \in S \text{ and } j \in V \setminus S\}$$

Integer program

Objective functions

$$\min \sum_{e \in E} c_e x_e$$

$$\min \sum_{k \in C} u_k$$

Constraints

$$
\begin{aligned}
\sum_{e \in \omega(\{i\})} x_e &= 2 && \forall i \in V \\
\sum_{e \in \omega(S)} x_e &\geq 2 && \forall S \subset V, 3 \leq |S| \leq |V| - 3 \\
x_e &\leq u_{\delta(e)} && \forall e \in E \\
x_e &\in \{0,1\} && \forall e \in E \\
u_k &\in \{0,1\} && \forall k \in C
\end{aligned}
$$

Valid constraints

$$u_k \leq \sum_{e \in \zeta(k)} x_e \quad \forall k \in C$$

State-of-the-art

Minimum labelling hamiltonian cycle problem (Tabu search) [Cerulli, Dell'Olmo, Gentili, Raiconi, 2006]

Colorful traveling salesman problem (heuristic, GA) [Xiong, Golden, Wasil, 2007]

Traveling salesman problem with labels (approximation algorithm) [Gourvès, Monnot, Telelis, 2008]

Minimum labelling spanning tree problem

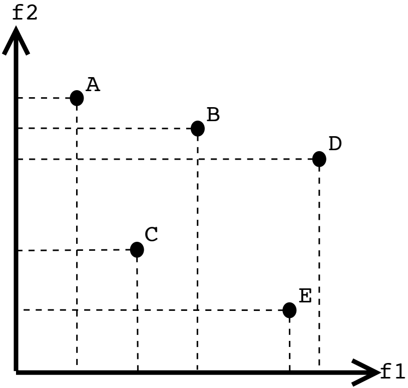
Multi-objective optimization problem

$$(PMO) = \begin{cases} min\ F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ s.t.\ x \in \Omega \end{cases}$$

with:

- $n \geq 2$: number of objectives
- $F = (f_1, f_2, \ldots, f_n)$: vector of functions to optimize
- $\Omega \subseteq \mathbb{R}^m$: set of feasible solutions
- $x = (x_1, x_2, \ldots, x_m) \in \Omega$: a feasible solution
- $\mathcal{Y} = F(\Omega)$: objective space
- $y = (y_1, y_2, \ldots, y_n) \in \mathcal{Y}$ avec $y_i = f_i(x)$: a point in the objective space

A solution x dominates ($\preceq$) a solution y if and only if
$\forall i \in \{1, \ldots, n\}, f_i(x) \leq f_i(y)$ and $\exists i \in \{1, \ldots, n\}$ such that $f_i(x) < f_i(y)$.



A multi-objective branch-and-cut algorithm

STEP 1 (Root of the tree)
Generate an initial upper bound ub
Define a first sub-problem
Insert the sub-problem in a list L

STEP 2 (Stopping criterion)
If $L = \emptyset$ then STOP, else choose a sub-problem from L and remove it from L

STEP 3 (Sub-problem solution)
Solve the sub-problem to obtain the lower bound lb

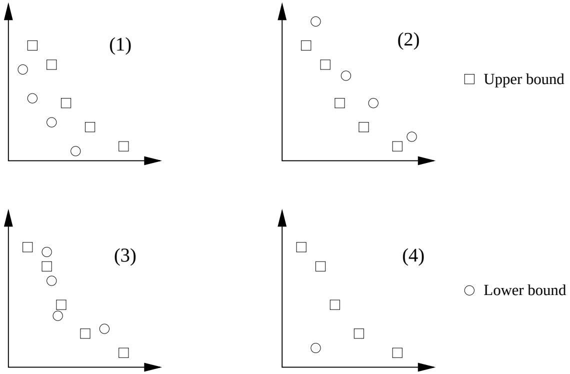
STEP 4 (Constraint generation)
If some integer solutions have been found, try to insert them in ub
if $ub \preceq lb$ **then**
  Go to STEP 2.
else
  if violated constraints are identified **then**
    Add them to the model and go to STEP 3.
  else
    Go to STEP 5.
  end if
end if

STEP 5 (Branching)
Branch on variable and introduce 2 new sub-problems in L. Go to STEP 2.

Adaptations to a multi-objective problem

Upper bound = set of non-dominated solutions found during the search

Lower bound = set of non-dominated points in the objective space such that all feasible solutions are dominated by these points



[Sourd et Spanjaard, 2008]

Computation of the lower bound

Initial sub-problem :

$$\min \quad \sum_{e \in E} c_e x_e$$

$$\min \quad \sum_{k \in C} u_k$$

$$\sum_{e \in \omega(\{i\})} x_e = 2 \qquad \forall i \in V$$

$$x_e \leq u_{\delta(e)} \qquad \forall e \in E$$

$$u_k \leq \sum_{e \in \zeta(k)} x_e \qquad \forall k \in C$$

$$0 \leq x_e \leq 1 \qquad \forall e \in E$$

$$0 \leq u_k \leq 1 \qquad \forall k \in C$$

Computation of the lower bound

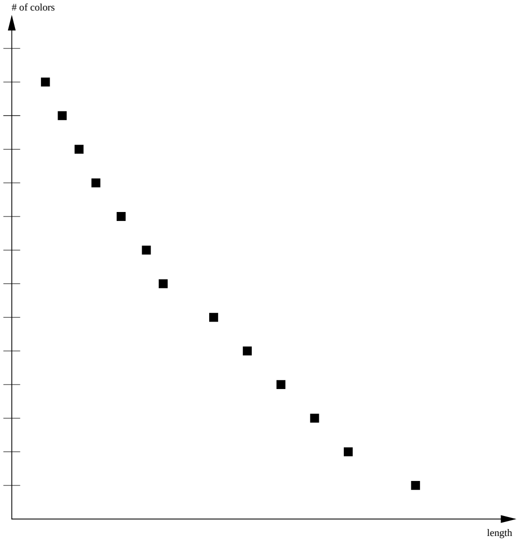
Solve the following problem for different values of ϵ

$$
\begin{aligned}
\min \sum_{e \in E} c_e x_e &+ m \sum_{k \in C} u_k & \\
\sum_{e \in \omega(\{i\})} x_e &= 2 & \forall i \in V \\
x_e &\leq u_{\delta(e)} & \forall e \in E \\
u_k &\leq \sum_{e \in \zeta(k)} x_e & \forall k \in C \\
\sum_{k \in C} u_k &\leq \epsilon & \\
0 \leq x_e &\leq 1 & \forall e \in E \\
0 \leq u_k &\leq 1 & \forall k \in C
\end{aligned}
$$

After founding non-dominated solution for a given ϵ, identify violated constraints and add them

Computation of the lower bound

Computation of the lower bound



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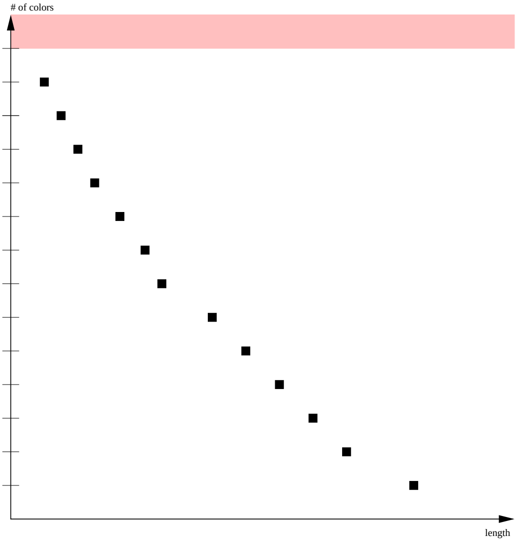

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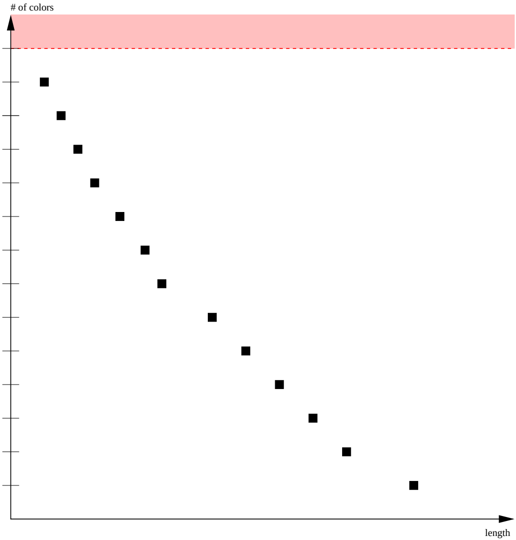

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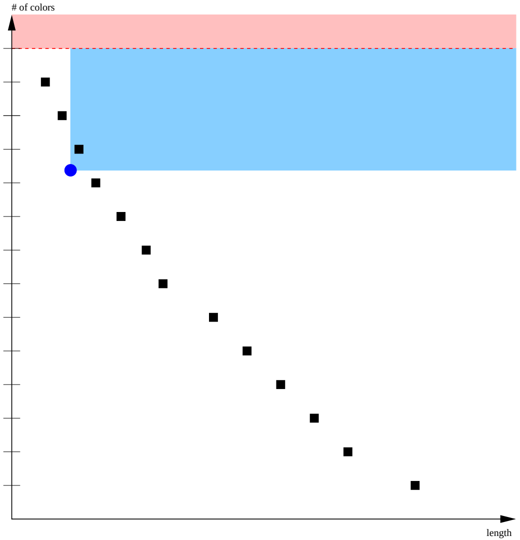

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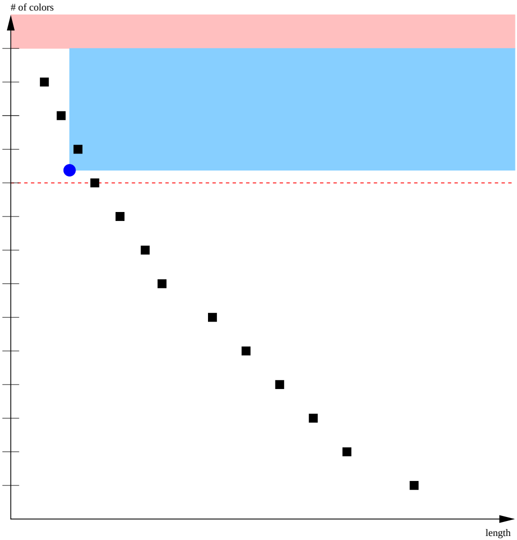

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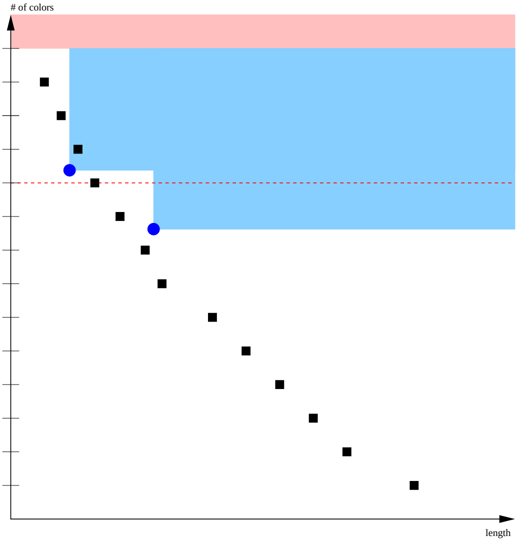

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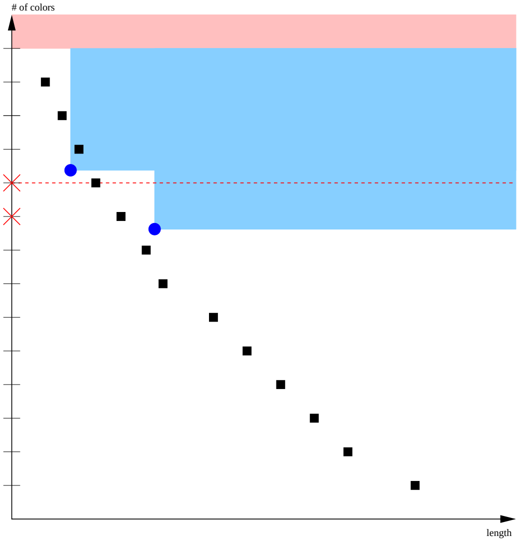

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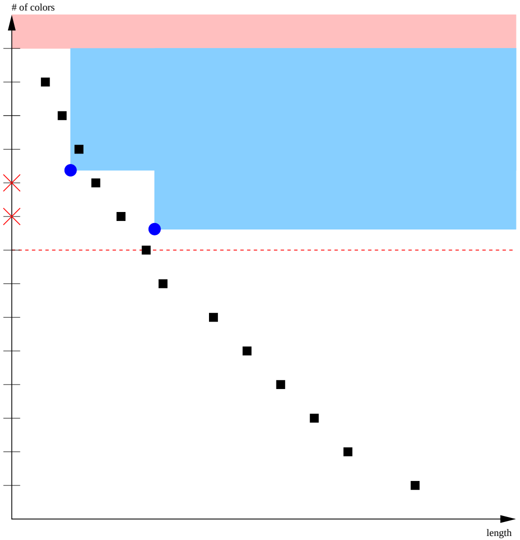

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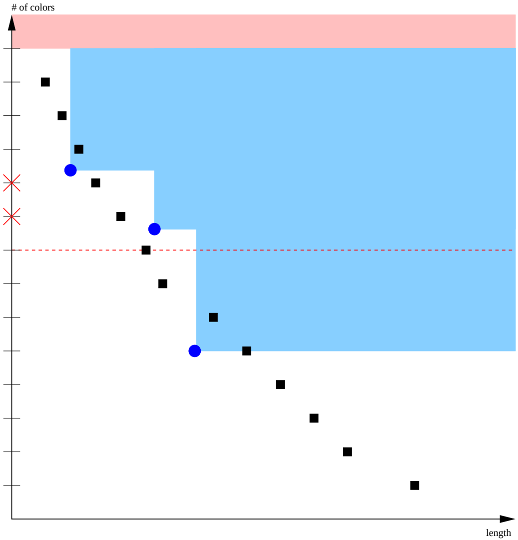

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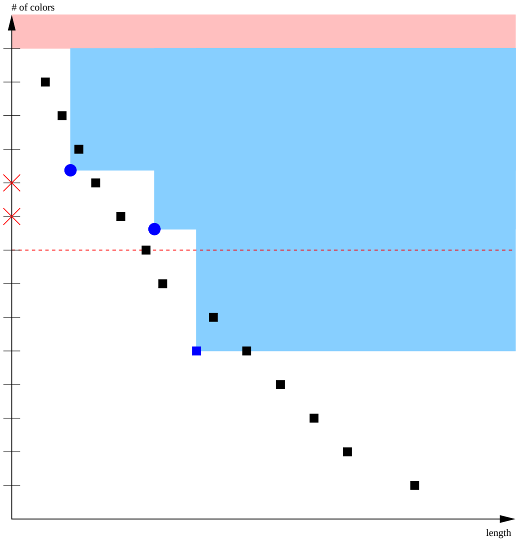

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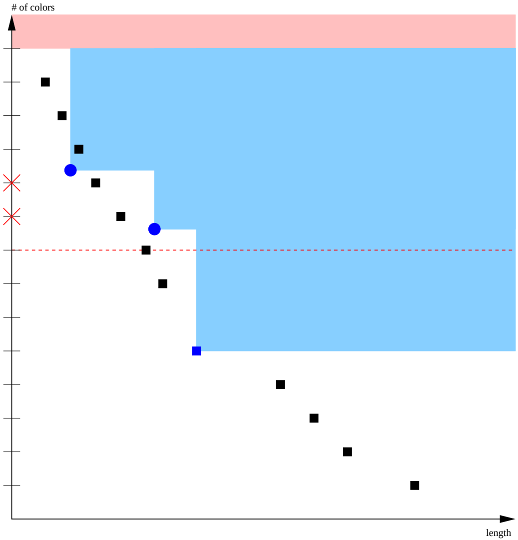

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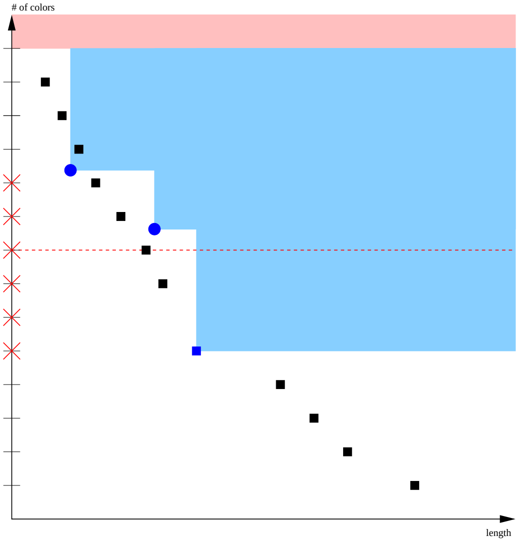

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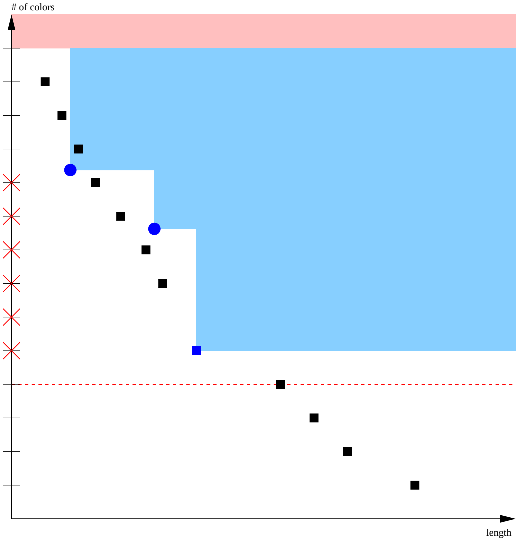

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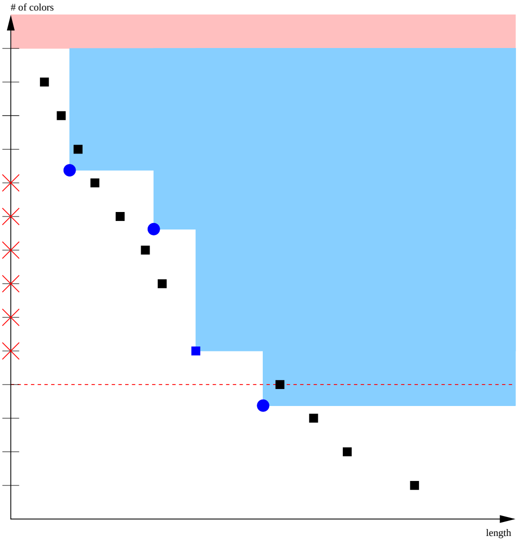

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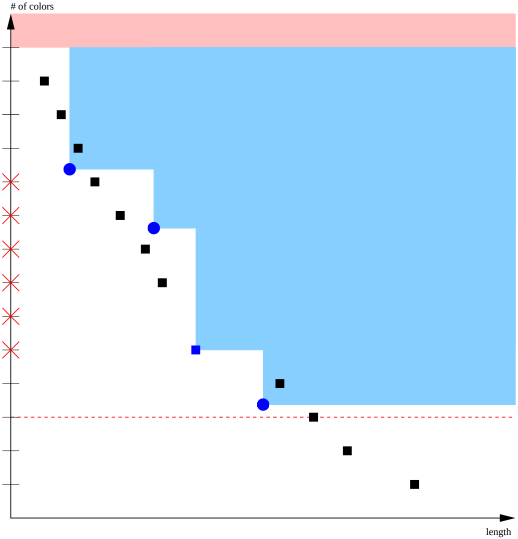

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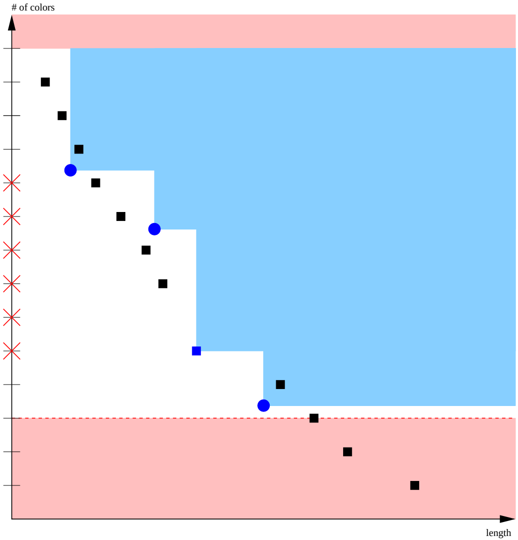

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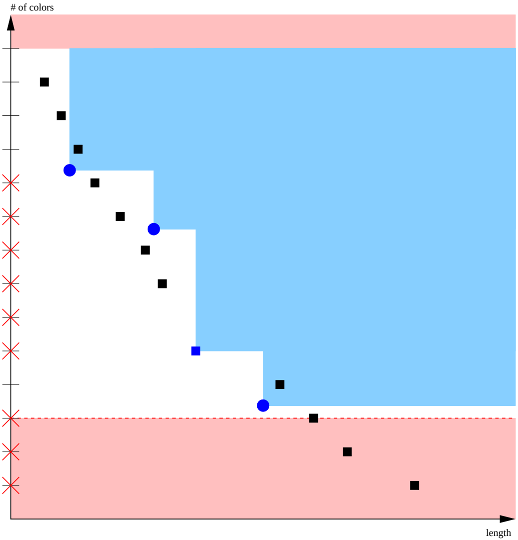

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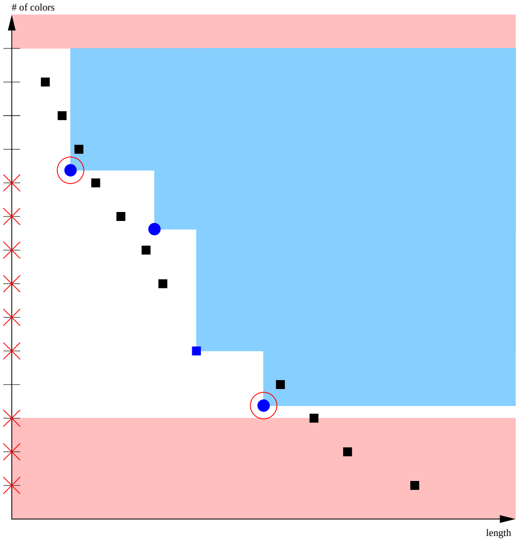

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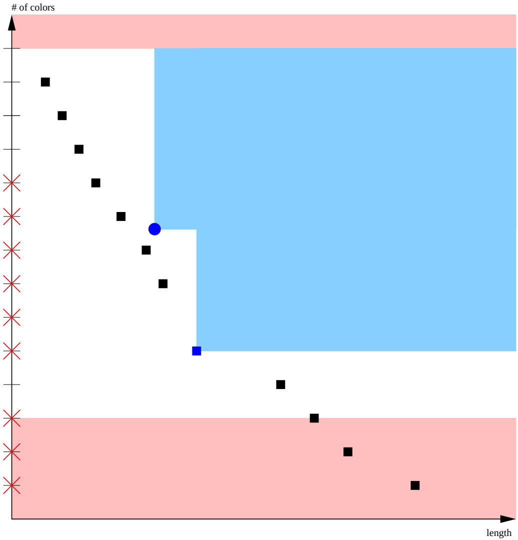

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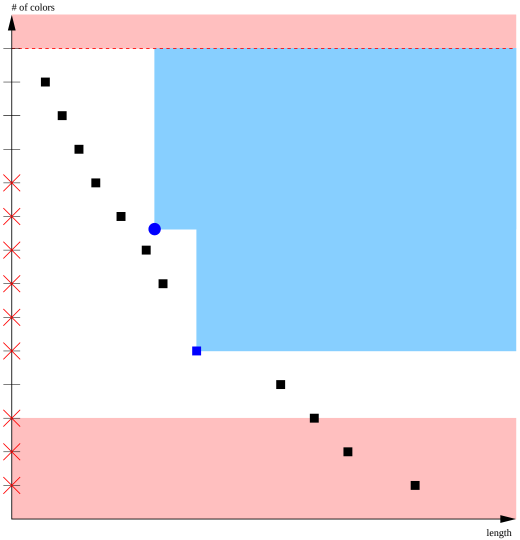

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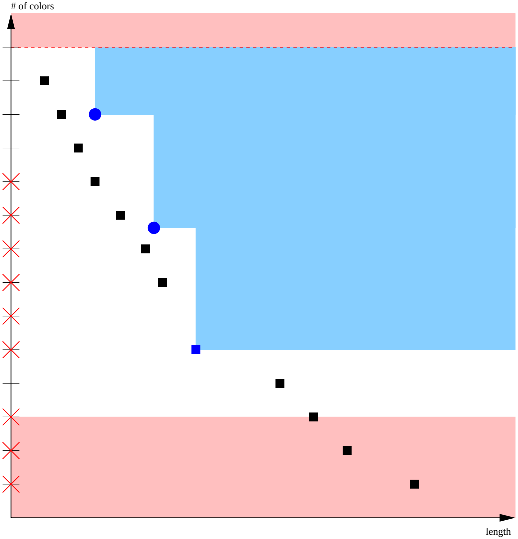

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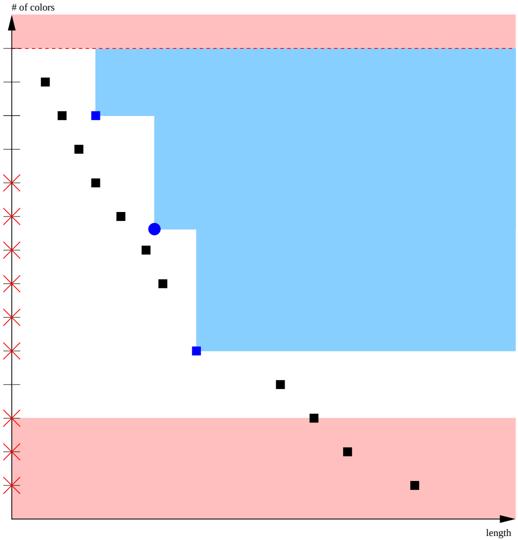

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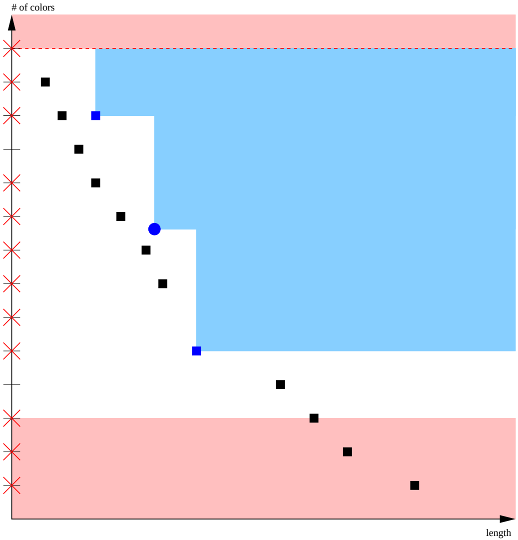

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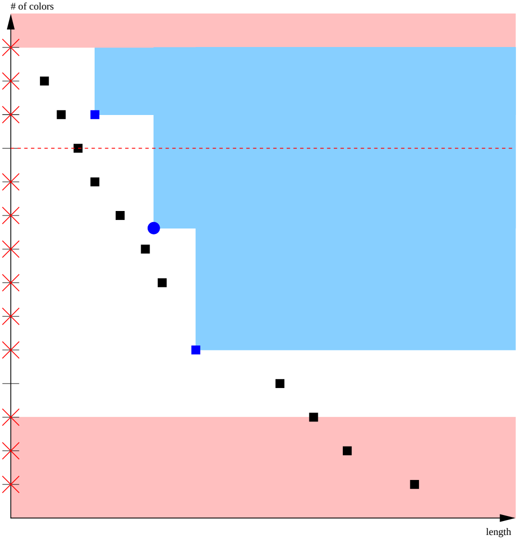

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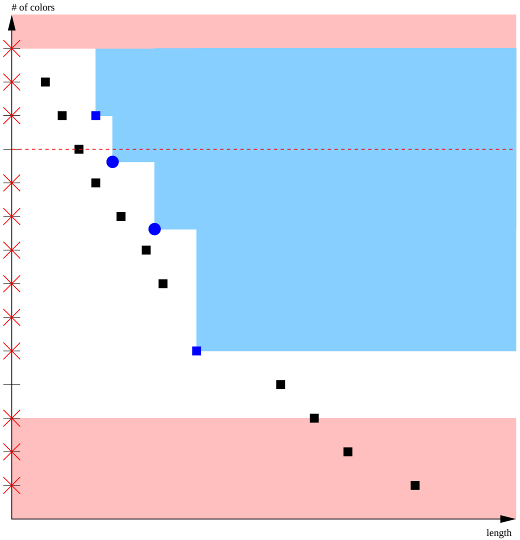

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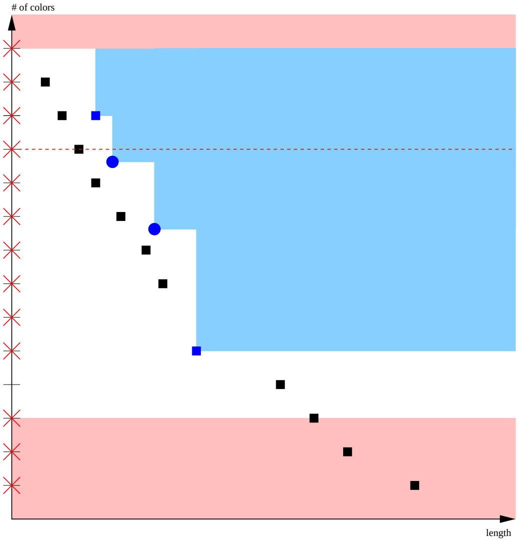

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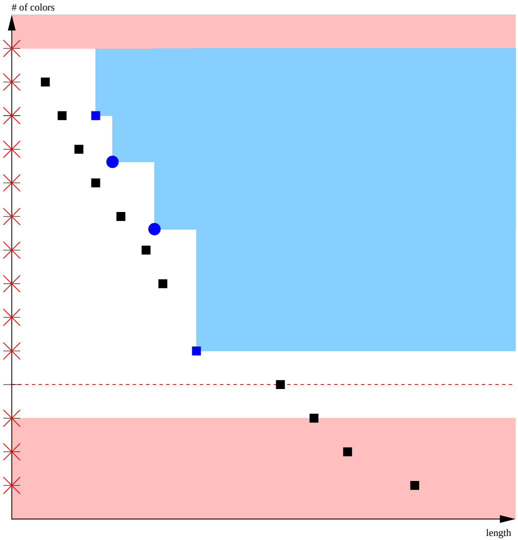

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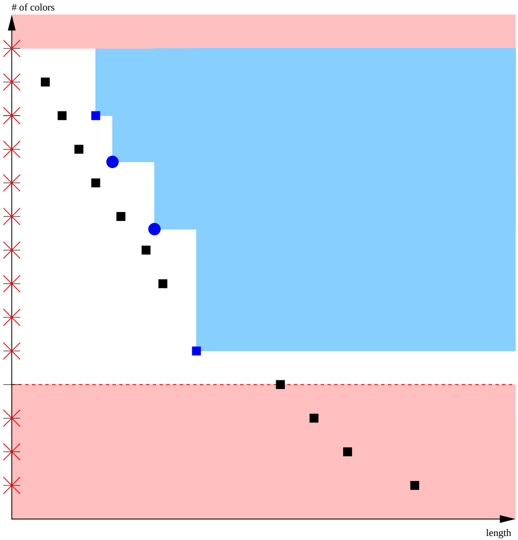

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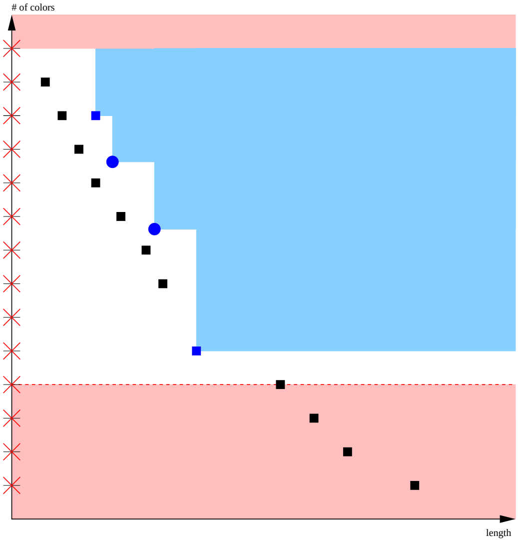


Constraint generation, cutting, and branching

Constraint generation

Connectivity constraints → min-cut problem

Call to a CONCORDE function [Padberg & Rinaldi, 1990]

Cutting

$\forall \epsilon$, the sub-problem is unfeasible

$\forall \epsilon$, the solution is either feasible or dominated by ub

Branching

First on the u_k variables then on the x_e

Priority on the variable that is non integral for the most values of ϵ

Computation of the initial upper bound

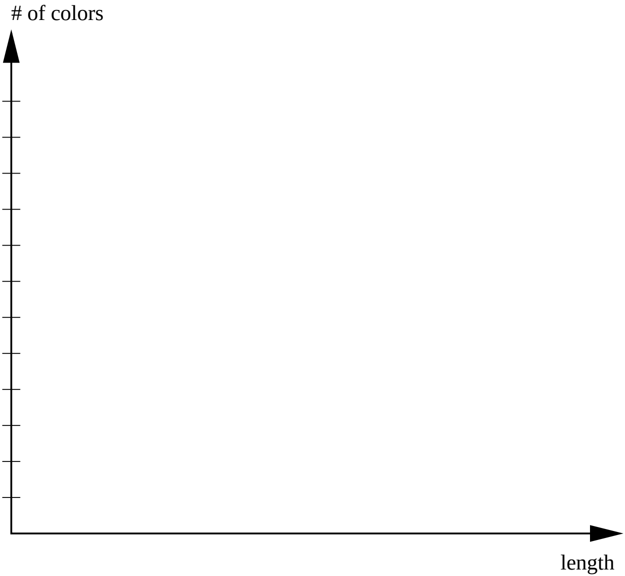
Repeat the following sequence for different values of ϵ

STEP 1: Solve the following mixed integer program :

$$
\begin{array}{rcll}
\min \displaystyle\sum_{e \in E} c_e x_e & + & m \displaystyle\sum_{k \in C} u_k & \\
\displaystyle\sum_{e \in \omega(\{i\})} x_e & = & 2 & \forall i \in V \\
x_e & \leq & u_{\delta(e)} & \forall e \in E \\
u_k & \leq & \displaystyle\sum_{e \in \zeta(k)} x_e & \forall k \in C \\
\displaystyle\sum_{k \in C} u_k & \leq & \epsilon & \\
0 \leq & x_e & \leq 1 & \forall e \in E \\
u_k & \in & \{0,1\} & \forall k \in C
\end{array}
$$

STEP 2: Solve a TSP on $G' = (V, E')$ with $E' = \{e \in E | u_{\delta(e)} = 1\}$

Computation of the initial upper bound



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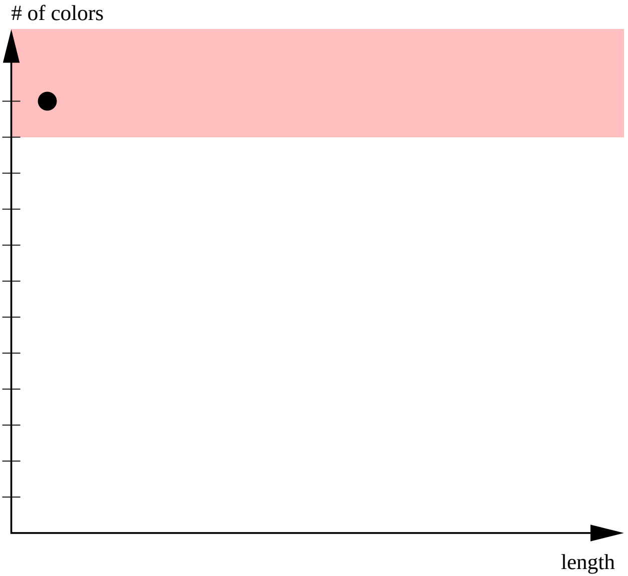

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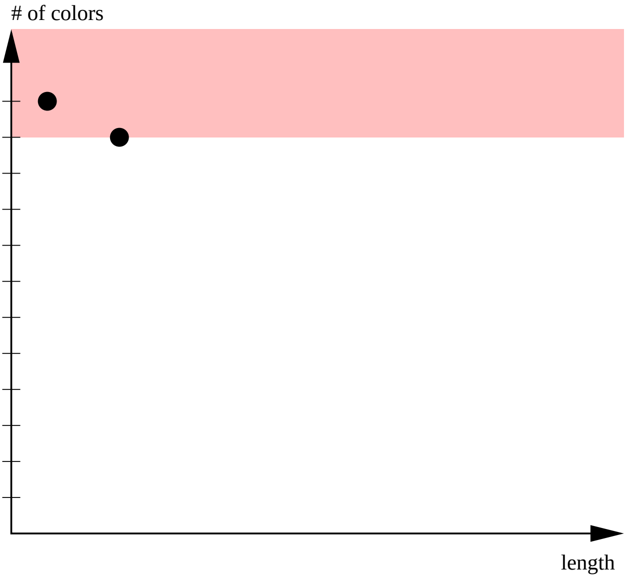

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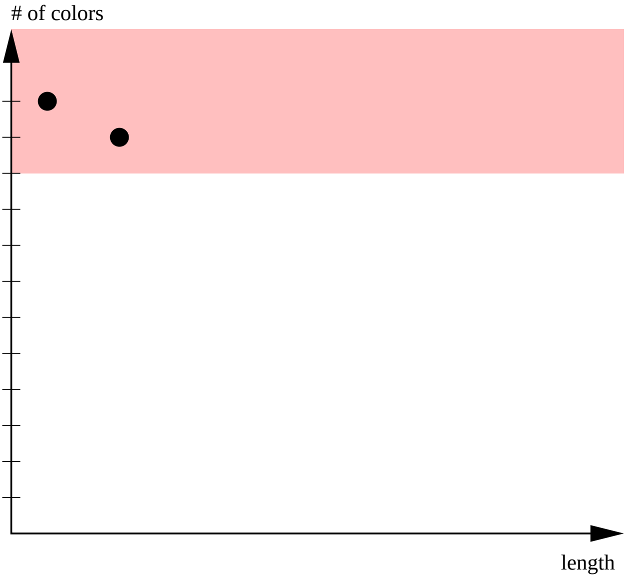

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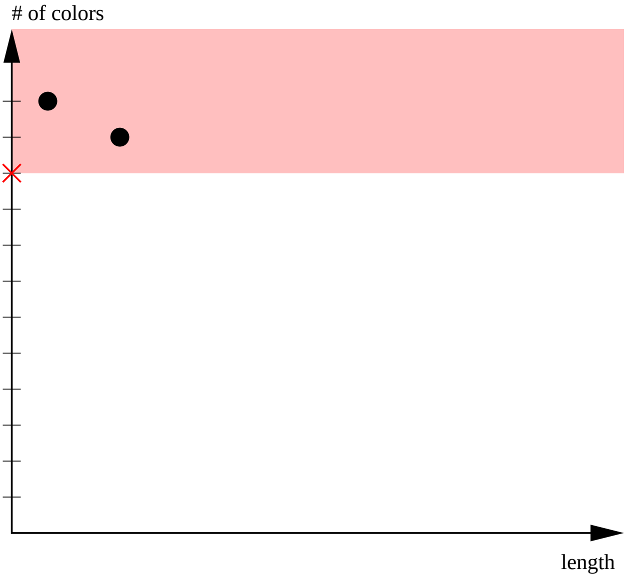

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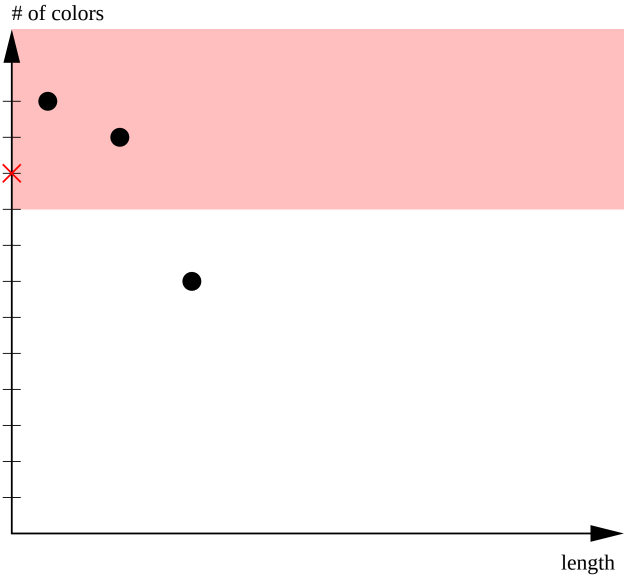

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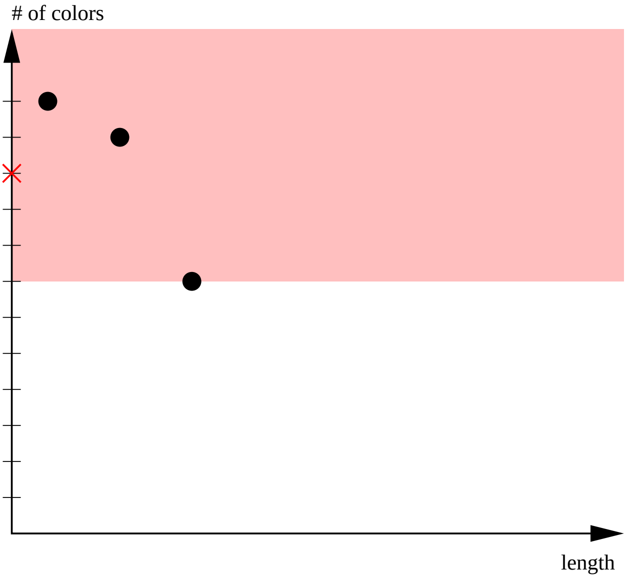

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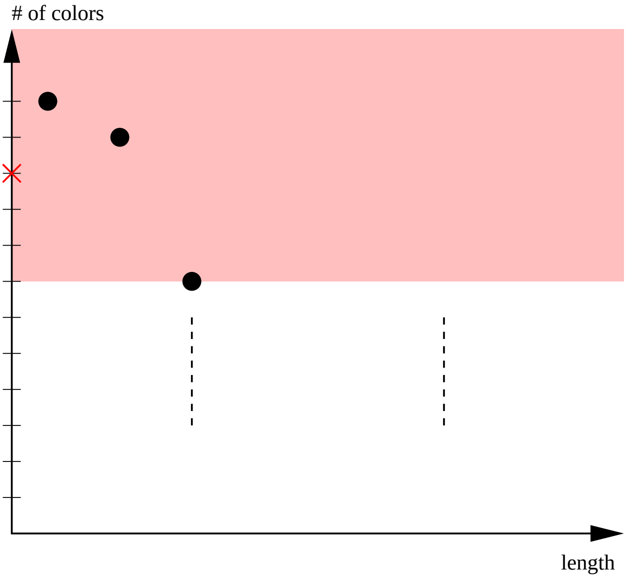

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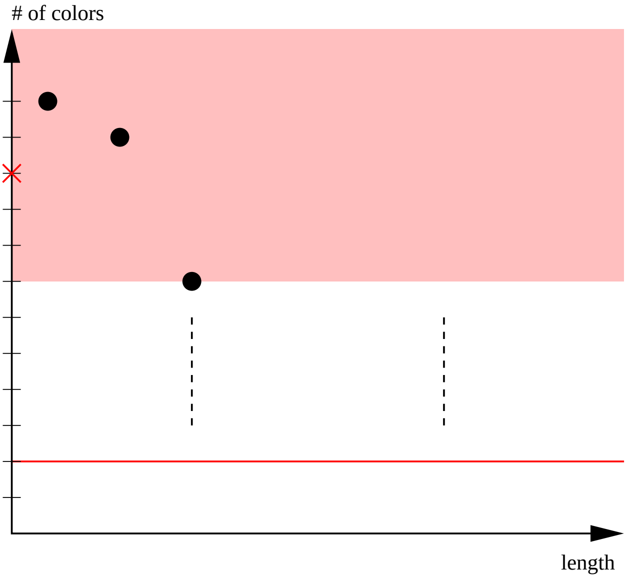

Computational results

$\lvert C \rvert$	$\lvert V \rvert$	$\times$	#nodes	#u	#x	#cut	#Pareto	#ub	#Parub
10	20	25	84	36	5	49	7	7	4
10	30	25	134	51	14	105	8	8	5
10	40	25	231	65	50	156	9	9	5
10	50	25	371	73	111	214	9	9	6
15	20	25	178	82	6	73	9	8	4
15	30	17	428	186	26	193	12	11	4
15	40	20	573	217	68	286	12	12	5
15	50	13	1102	298	252	432	13	13	5
20	20	25	372	180	5	123	10	9	4
20	30	25	815	376	31	274	13	12	4
20	40	25	1489	653	91	477	15	14	5
20	50	25	2468	898	335	759	17	16	5
25	20	25	466	226	5	130	11	9	4
25	30	25	1564	755	26	412	15	12	4
25	40	25	3244	1455	166	788	17	15	4
25	50	15	3756	1627	249	988	20	18	5
30	20	25	814	401	4	199	12	9	4
30	30	25	2274	1089	47	490	16	13	4
30	40	25	4875	2254	182	966	18	16	4
30	50	6	7752	3659	215	1563	20	17	4

Computational results

$\lvert C \rvert$	$\lvert V \rvert$	$\times$	time(bc)	time(ub)	time
10	20	25	1	0	1
10	30	25	11	0	12
10	40	25	39	2	41
10	50	25	112	5	117
15	20	25	4	0	5
15	30	17	50	1	52
15	40	20	165	5	171
15	50	13	527	12	538
20	20	25	12	0	13
20	30	25	142	3	146
20	40	25	731	8	739
20	50	25	3051	20	3071
25	20	25	17	0	17
25	30	25	392	4	397
25	40	25	3362	13	3376
25	50	15	8243	31	8268
30	20	25	36	0	37
30	30	25	749	5	754
30	40	25	5212	17	5229
30	50	6	35001	43	35045

Conclusions and perspectives

- Branch-and-cut algorithm able to solve a multi-objective problem in one run
- Identify new valid constraints $\rightarrow$ variables u_k
- Rules to choose on which variables to branch
- Progressive partition of the objective space