A branch-and-bound algorithm for a travelling salesman problem with colors and two objectives

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Multi-objective optimization problem

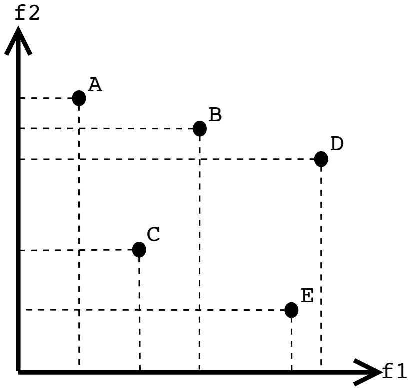
$$(PMO) = \begin{cases} min\ F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ s.t.\ x \in \Omega \end{cases}$$

with:

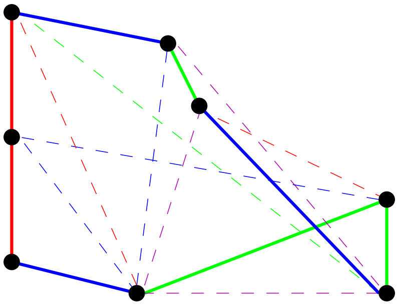
- $n \geq 2$: number of objectives
- $F = (f_1, f_2, \ldots, f_n)$: vector of functions to optimize
- $\Omega \subseteq \Re^m$: set of feasible solutions
- $x = (x_1, x_2, \ldots, x_m) \in \Omega$: a feasible solution
- $\mathcal{Y} = F(\Omega)$: objective space
- $y = (y_1, y_2, \ldots, y_n) \in \mathcal{Y}$ avec $y_i = f_i(x)$: a point in the objective space

Pareto domination

A solution x dominates ($\preceq$) a solution y if and only if
$\forall i \in \{1, \ldots, n\}, f_i(x) \leq f_i(y)$ and $\exists i \in \{1, \ldots, n\}$ such that $f_i(x) < f_i(y)$.



The problem



Find a Hamiltonian cycle

Two objectives:

1. Minimize the total length of the cycle
2. Minimize the number of colors appearing on the cycle

State-of-the-art

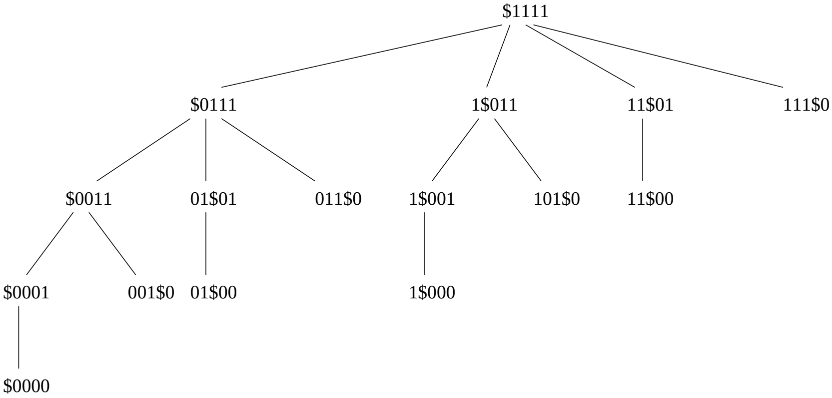
Minimum labelling hamiltonian cycle problem (Tabu search) [Cerulli, Dell'Olmo, Gentili, Raiconi, 2006]

Colorful traveling salesman problem (heuristic, GA) [Xiong, Golden, Wasil, 2007]

Traveling salesman problem with labels (approxitmation algorithm) [Gourvès, Monnot, Telelis, 2008]

Minimum labelling spanning tree problem

The branch-and-bound algorithm



One node = a set of possible colors

The cycle is obtained by CONCORDE

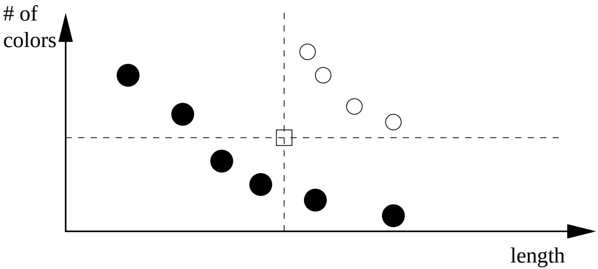
Upper bound: archive of non dominated solutions

Pruning

Infeasiblily: no Hamiltonian cycle
- Subset of an unfeasible subset of colors (list)
- Non connectivity
- Failure of the lowerbound computation
- CONCORDE

Branching is only done on colors that appears in the optimal tour found by CONCORDE

The ideal point (lower bound) is dominated by at least a solution of the archive (upperbound)



Lower bounds

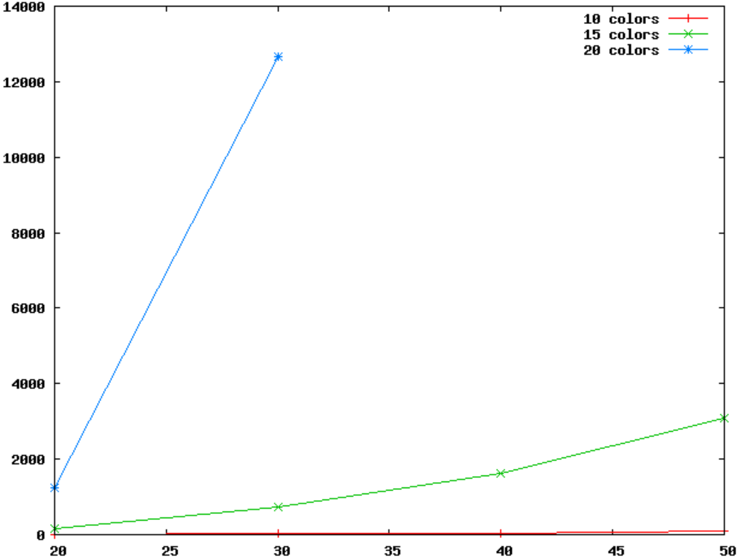
Total length

Length of the optimal tour in the parent node

Number of colors

$$\begin{cases} \min \sum_{c_j \in C} y_j \\ \text{s.t.} \\ \sum_{c_j \in C} \delta_{ij} y_j \geq 2, \ \forall v_i \in V \\ y_j \in \{0, 1\}, \ \forall c_j \in C \end{cases}$$

Computational times



Computational results

$\lvert C \rvert$	$\lvert V \rvert$	max # nodes	# nodes	# sol	# not conn.	# in list	# not mip lb	# CONC	# dom
10	20	1024	769	7	2	0	93	0	38
10	30	1024	893	8	1	0	67	0	36
10	40	1024	919	9	0	0	83	0	27
10	50	1024	962	9	0	0	97	0	17
15	20	32768	12156	9	9	0	273	1	871
15	30	32768	22217	13	13	0	460	0	536
15	40	32768	26669	13	2	0	667	0	393
15	50	32768	30415	14	0	0	578	0	212
20	20	1048576	85738	10	94	0	1994	9	6075
20	30	1048576	357176	14	55	0	3993	1	5235

Conclusions

- Better lower bounds (colors and distance)
- Better upper bound at the start → heuristic (efficient, relevant to the objective space)
- Piece-wise approximation for the lower bound ([Sourd & Spanjaard, 2008])
- Implementation
- Meta-heuristics
- Cooperation