Limited Discrepancy Search for flexible shop scheduling

Búsqueda de discrepancia limitada para programación de taller flexible

Pierre Lopez

LAAS-CNRS

Toulouse, France

(joint work with A. Ben Hmida, M.-J. Huguet, M. Haouari)

II SEMINARIO DE LOGÍSTICA INTEGRAL

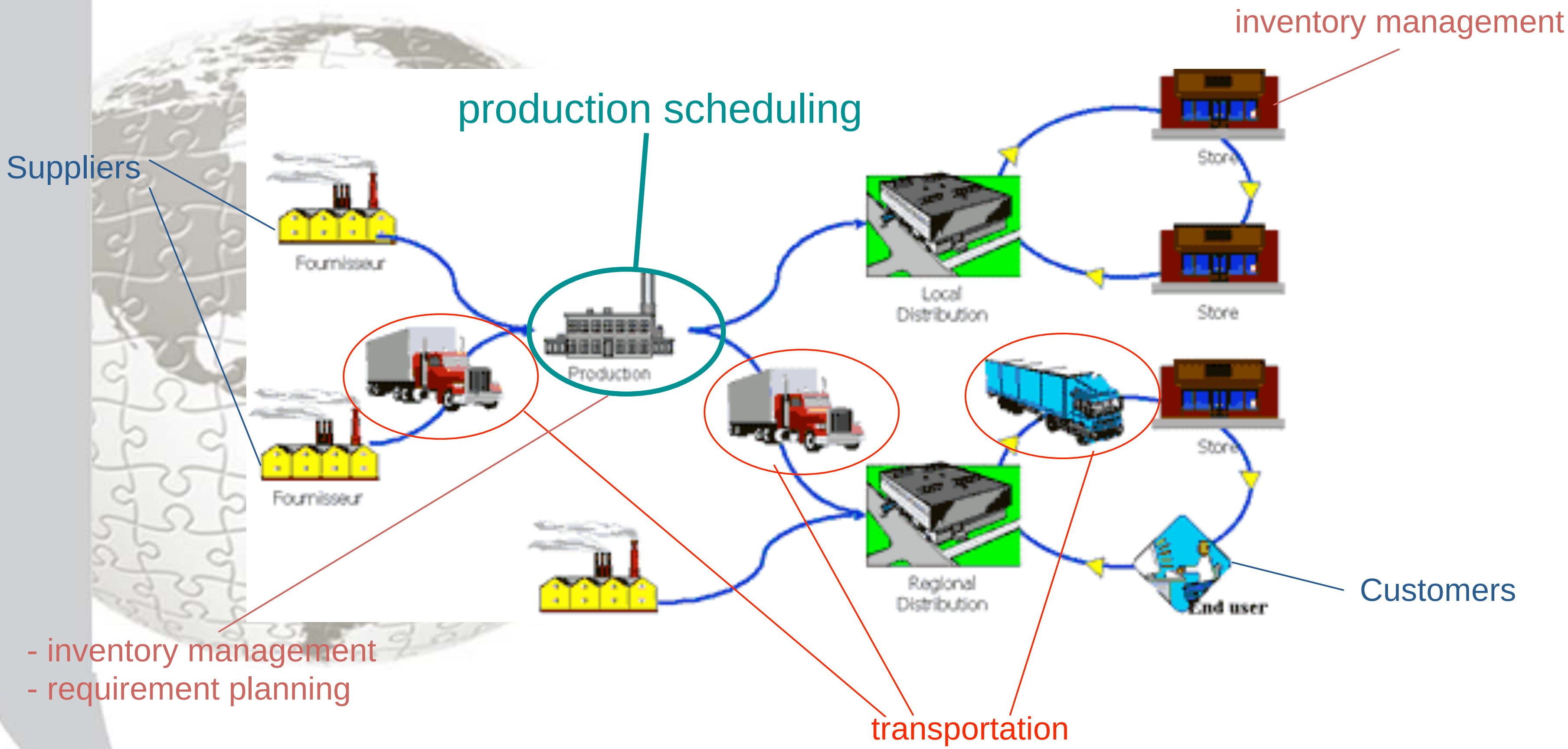
19 y 20 de Febrero de 2009

Objectives

- Some **research results** in solving **scheduling problems**, central in supply chain management
- **Technical** presentation!
- Emphasis on **flexible shop problems**
- Adapting **discrepancy-based search methods** for the problems under study
- **Experimental evaluation** of the propositions

Scheduling at the core of SCM

One of the main activities in Supply Chain Management



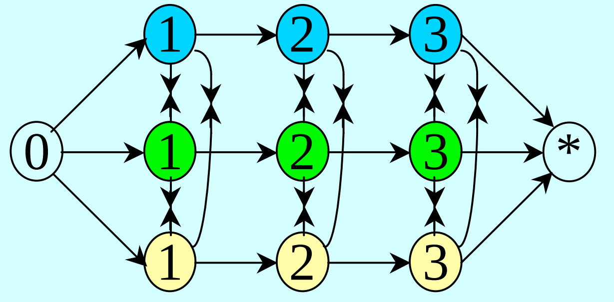
Hierarchical organization in production management

strategic	project management	~ year
tactical	mid-term planning	~ month
operational	scheduling	~ week / day

production system

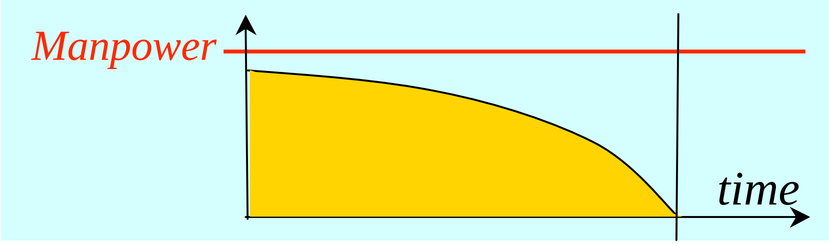
Scheduling Problems

- Scheduling with disjunctive resources
  - Shop / Processor scheduling
    - one-machine
    - *m* parallel machines
    - *m* dedicated machines
  - Timetabling

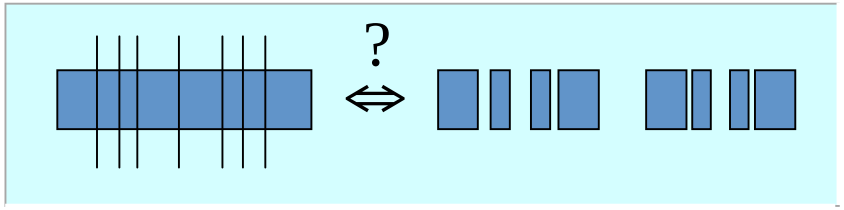


A 3×3 *flow-shop problem*

- Scheduling with cumulative resources
  - renewable/non renewable
  - doubly-constrained resources



- Preemptive/non preemptive scheduling



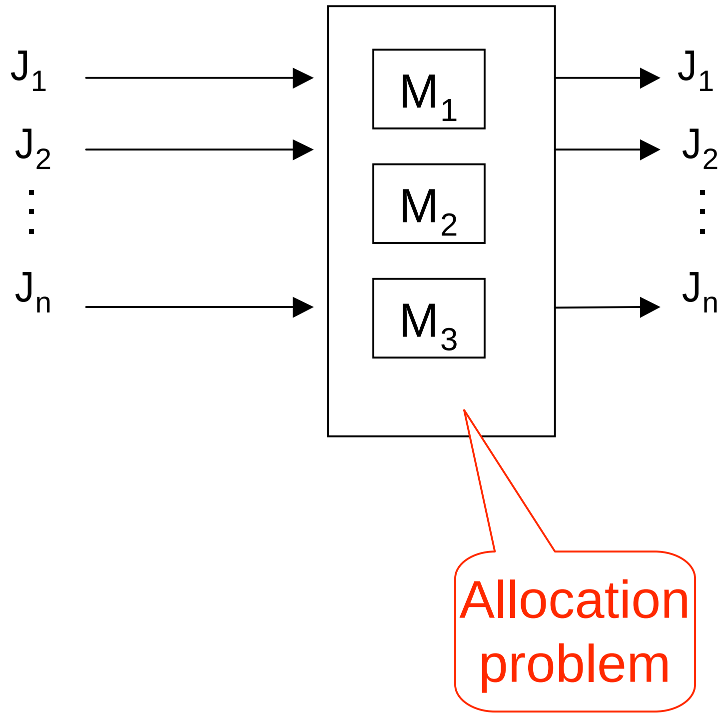
Universidad de los Andes

Scheduling Problems under study

- Disjunctive scheduling
  - Resources = machines
  - One operation at a time on a machine
  - One machine at a time by operation
  - No preemption
    - ➢ Shop problems
      - Execution route (routing) = sequence of machines to follow to manufacture a product
      - Job = sequence of operations following a given route

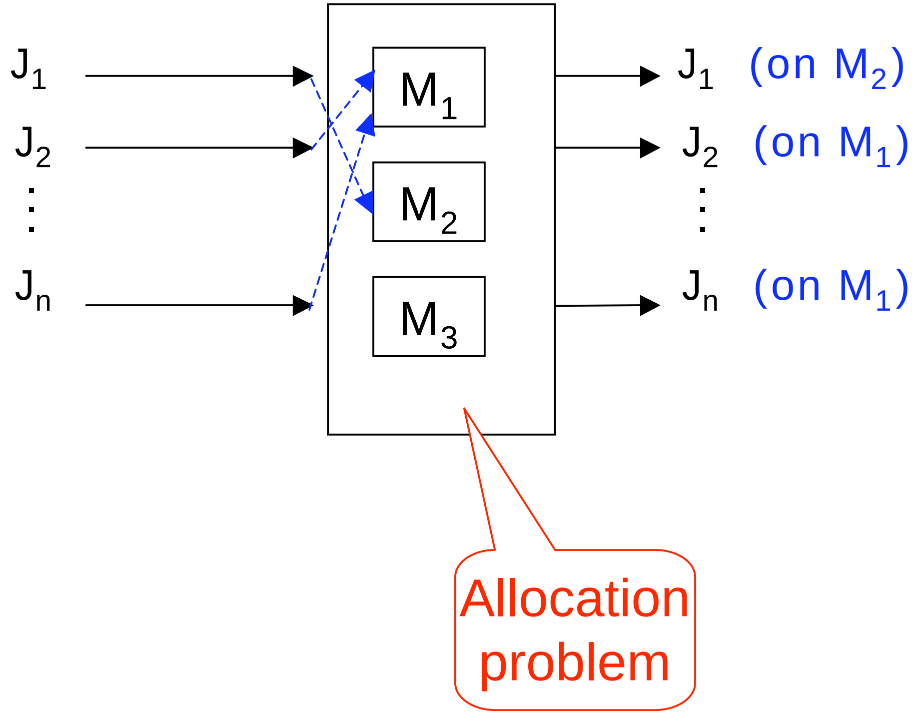
- Focus on *flexible* shop problems
  - Resource assignment is **not** decided *a priori*
    - ➢ Parallel Machine
    - ➢ Hybrid Flow Shop (HFS)
    - ➢ Flexible Job Shop (FJS)

Parallel machine



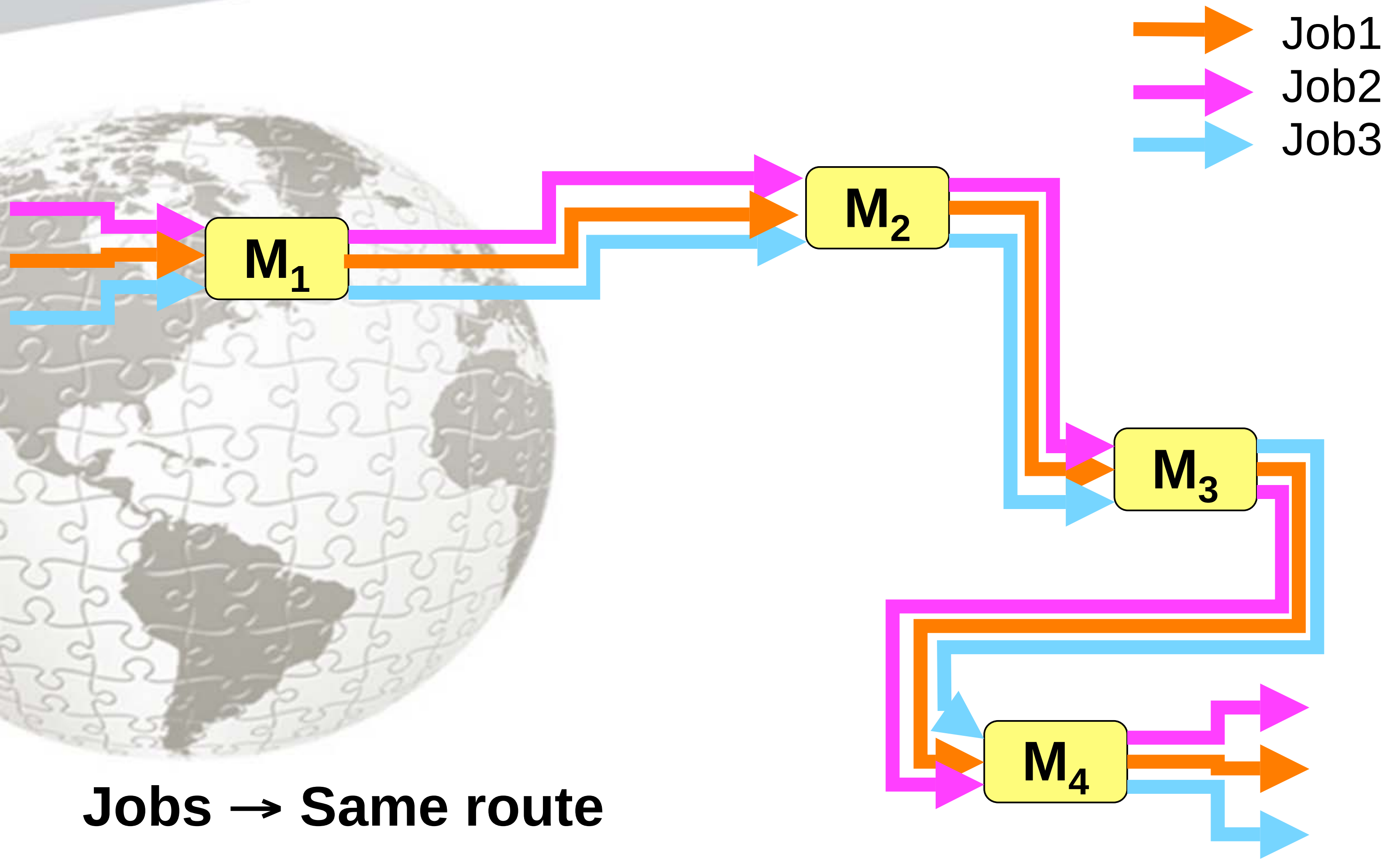
Job ≡ Operation

Parallel machine



Job ≡ Operation

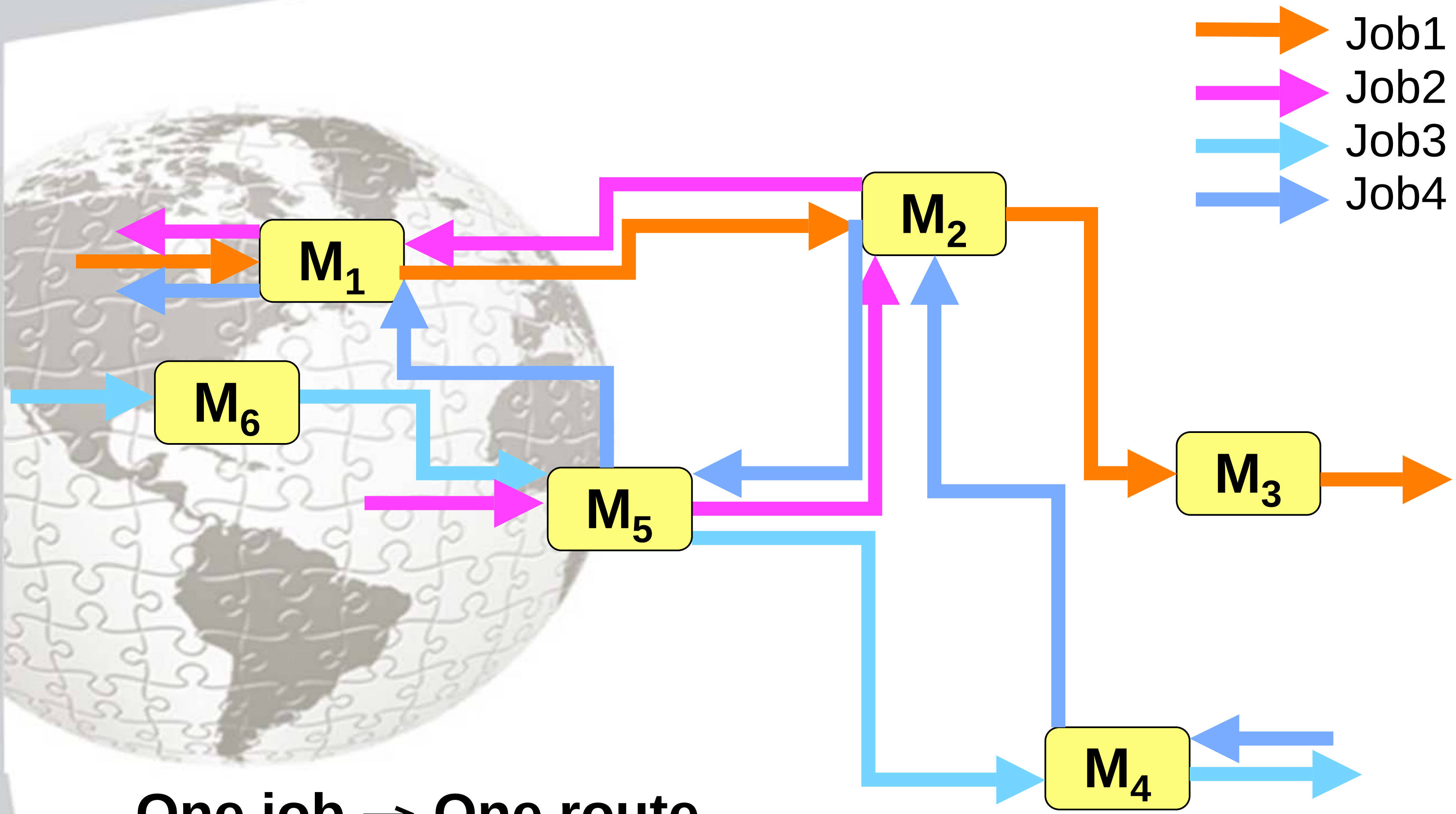
Shop problems: Flow Shop



Jobs → Same route

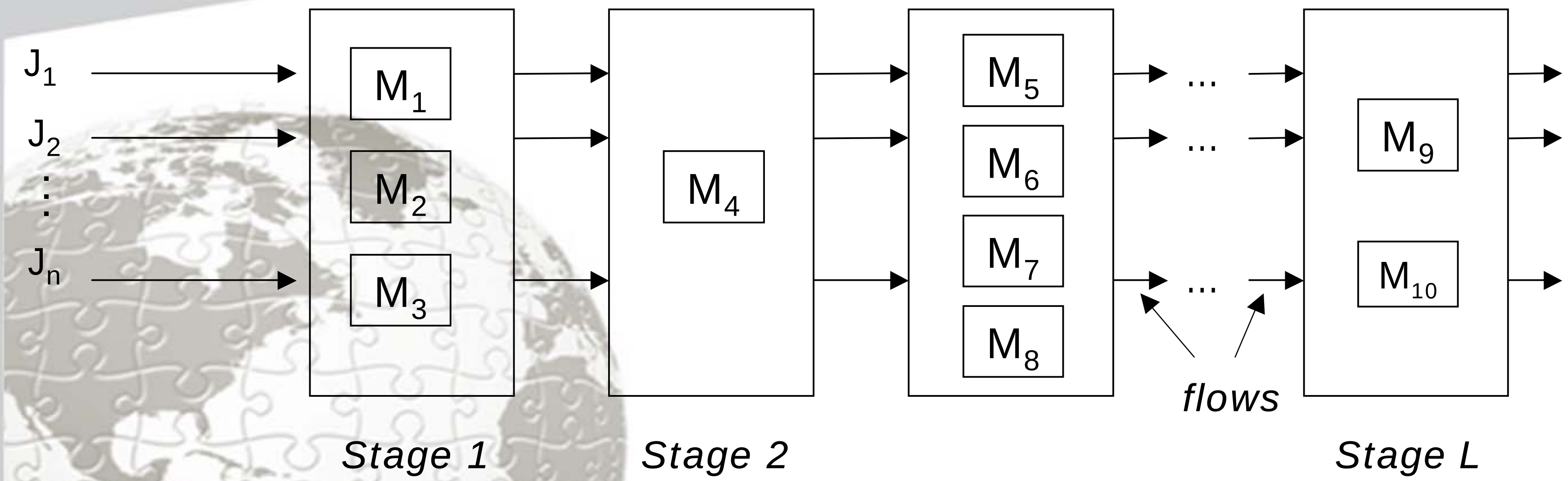
Universidad de los Andes

Shop problems: Job Shop



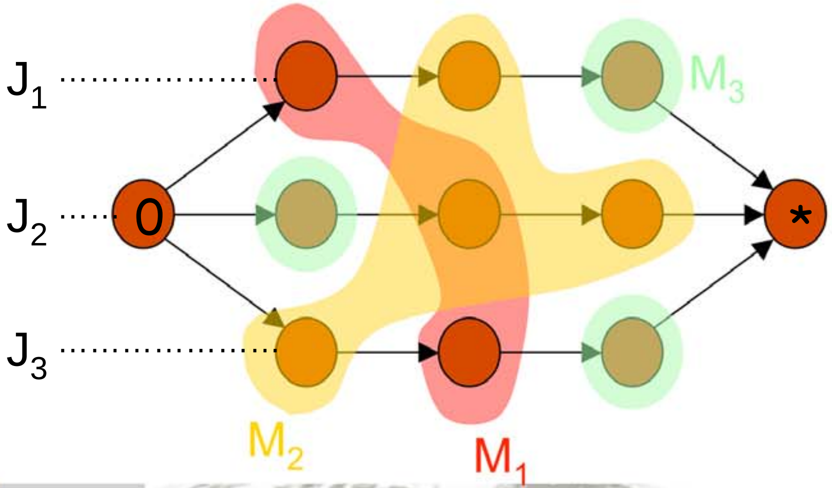
One job → One route

Shop problems: Hybrid Flow Shop



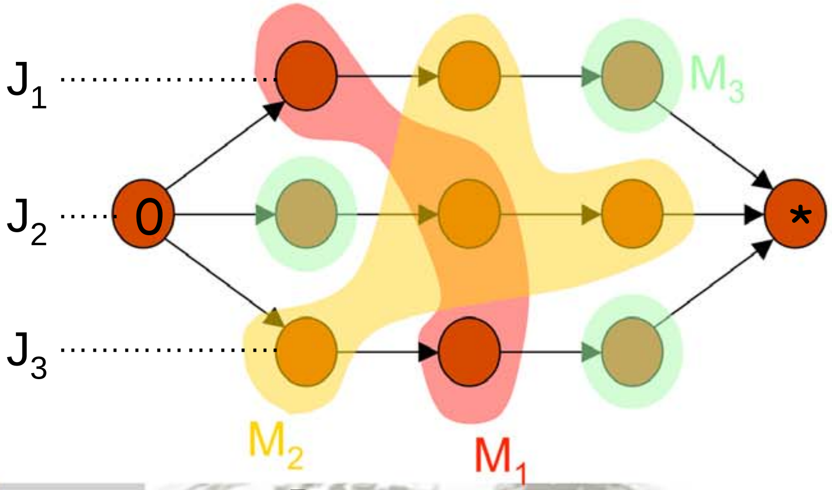
- J={J_1,…,J_i,…,J_n} jobs
- E={1,…,s,…, L} stages
- $M^{(s)}$: identical parallel machines / max ($M^{(s)}$)>1
- Application: semiconductors

  (Printed Circuit Boards)



Shop pbs: Flexible Job Shop

- As in a Job Shop:
  - Resources = machines: $M=\{M_1,\ldots,M_m\}$
  - Job J_i= sequence of operations $O_{i1},\ldots,O_{im}$



Shop pbs: Flexible Job Shop

- As in a Job Shop:
  - Resources = machines: $M=\{M_1,\ldots,M_m\}$
  - Job J_i= sequence of operations $O_{i1},\ldots,O_{im}$
- But:
  - Alternative (unrelated) machines can process an operation
    - O_{is} on any machine among $M_{is} \subseteq M$;
    - $\forall i, \cap M_{is}$ may be non-empty ("recirculation")

Shop pbs: Flexible Job Shop

	M_{21}	M_{22}	M_{23}
J_2:	$M_1(20)$ or $M_2(25)$	$M_1(25)$	$M_1(30)$ or $M_2(15)$ or $M_3(25)$

- As in a Job Shop:
  - Resources = machines: $M=\{M_1,\ldots,M_m\}$
  - Job J_i= sequence of operations $O_{i1},\ldots,O_{im}$
- But:
  - Alternative (unrelated) machines can process an operation
    - O_{is} on any machine among $M_{is} \subseteq M$;
    - $\forall i, \cap\, M_{is}$ may be non-empty ("recirculation")

Shop pbs: Flexible Job Shop

- As in a Job Shop:
  - Resources = machines: $M=\{M_1,\ldots,M_m\}$
  - Job J_i= sequence of operations $O_{i1},\ldots,O_{im}$

	M_{21}	M_{22}	M_{23}
J_2:	$M_1(20)$ or $M_2(25)$	$M_1(25)$	$M_1(30)$ or $M_2(15)$ or $M_3(25)$

- But:
  - Alternative (unrelated) machines can process an operation
    - O_{is} on any machine among $M_{is} \subseteq M$;
    - $\forall i, \cap M_{is}$ may be non-empty ("recirculation")
- Application: semiconductor industry (wafer fabrication)
- Double problem:
  - Select a machine for each operation
  - Determine a start time for each operation

$$\boxed{\min C_{max}}$$

Solving scheduling problems

- **Exact methods**
  - dynamic programming
  - integer programming
  - tree search
- **Heuristics**
  - dispatching rules
  - greedy algorithms
- **Metaheuristics**
  - Tabu search
  - genetic algorithms
  - ant colony optimization
- **Constraint programming**

Solving scheduling problems

- **Exact methods**
  - dynamic programming
  - integer programming
  - tree search
- **Heuristics**
  - dispatching rules
  - greedy algorithms
- **Metaheuristics**
  - Tabu search
  - genetic algorithms
  - ant colony optimization
- **Constraint programming**

Background

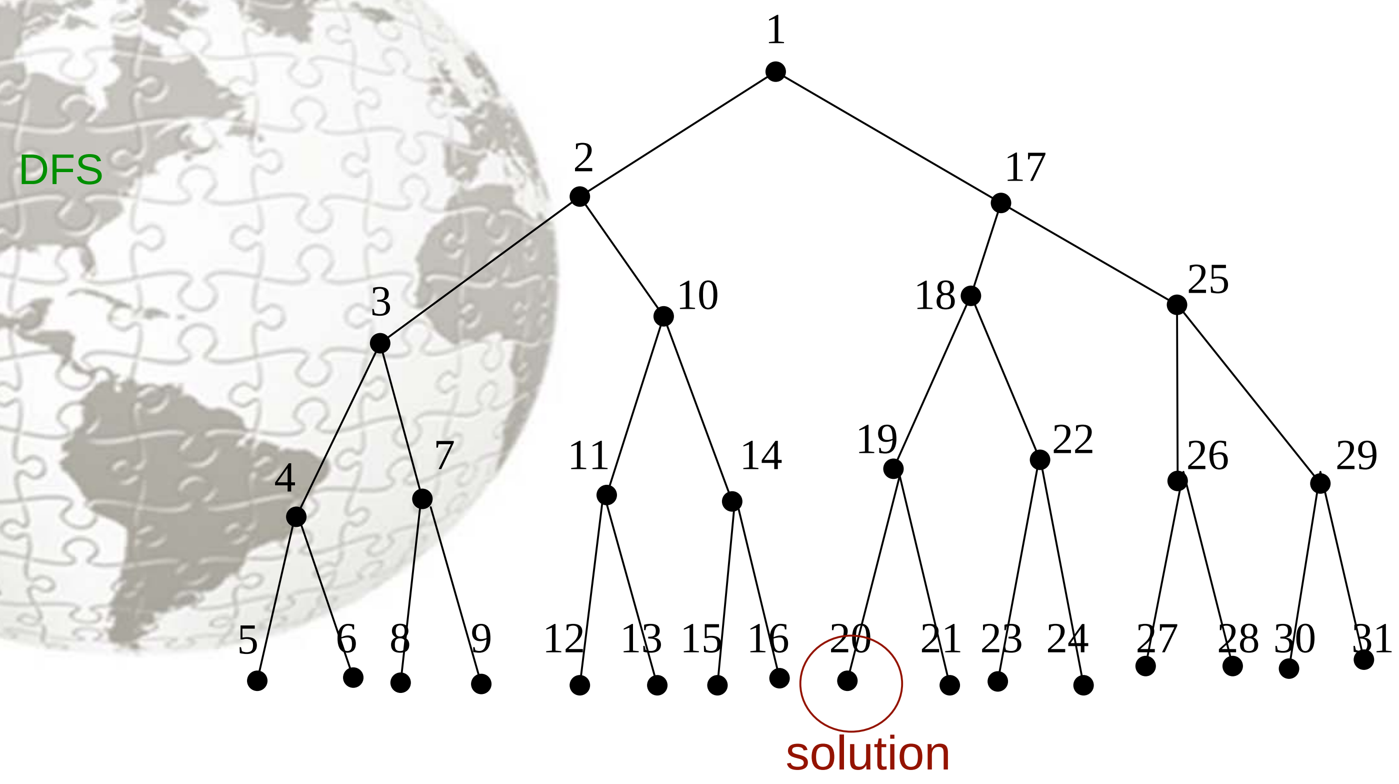
- *NP-hard* problems [Vaessens, 1995]
- Hybrid Flow Shop (HFS)
  - Exact methods: [Brah & Hunsucker, 1991]; [Portmann *et al*., 1992]; [Moursli & Pochet, 2000]; [Carlier & Néron, 2000]; [Lin & Liao, 2003]
  - Lower bounds: [Santos *et al*., 1995]; [Moursli & Pochet, 2000]; [Carlier & Néron, 2000]
  - Heuristics: [Brah & Loo, 1999]; [Engin & Döyen, 2004]
- Flexible Job Shop (FJS)
  - First presented by [Brucker & Schlie, 1990]
  - FJSP is NP-Hard in general [Vaessens, 1995]
  - Greedy and GA algorithms were proposed (many references)
  - Best results obtained by Tabu Search [Mastrolilli & Gambardella, 2000]
  - JMPM | | C_{max} is strongly *NP-hard* [Brucker, 2004]

Discrepancy-based search methods (1)

- Limited Discrepancy Search - LDS [Harvey & Ginsberg, 1995]
  - Is a problem satisfiable? → *Satisfaction*
  - Iterative tree search method
  - Instantiation heuristic to guide the search
    (the initial global instantiation is not necessarily a solution)
  - When the heuristic does not find a good solution, it is probably because it made a few poor choices → *discrepancy* then makes a choice different than heuristically top-ranked
  - Hope to find a solution before *Depth-First Search*

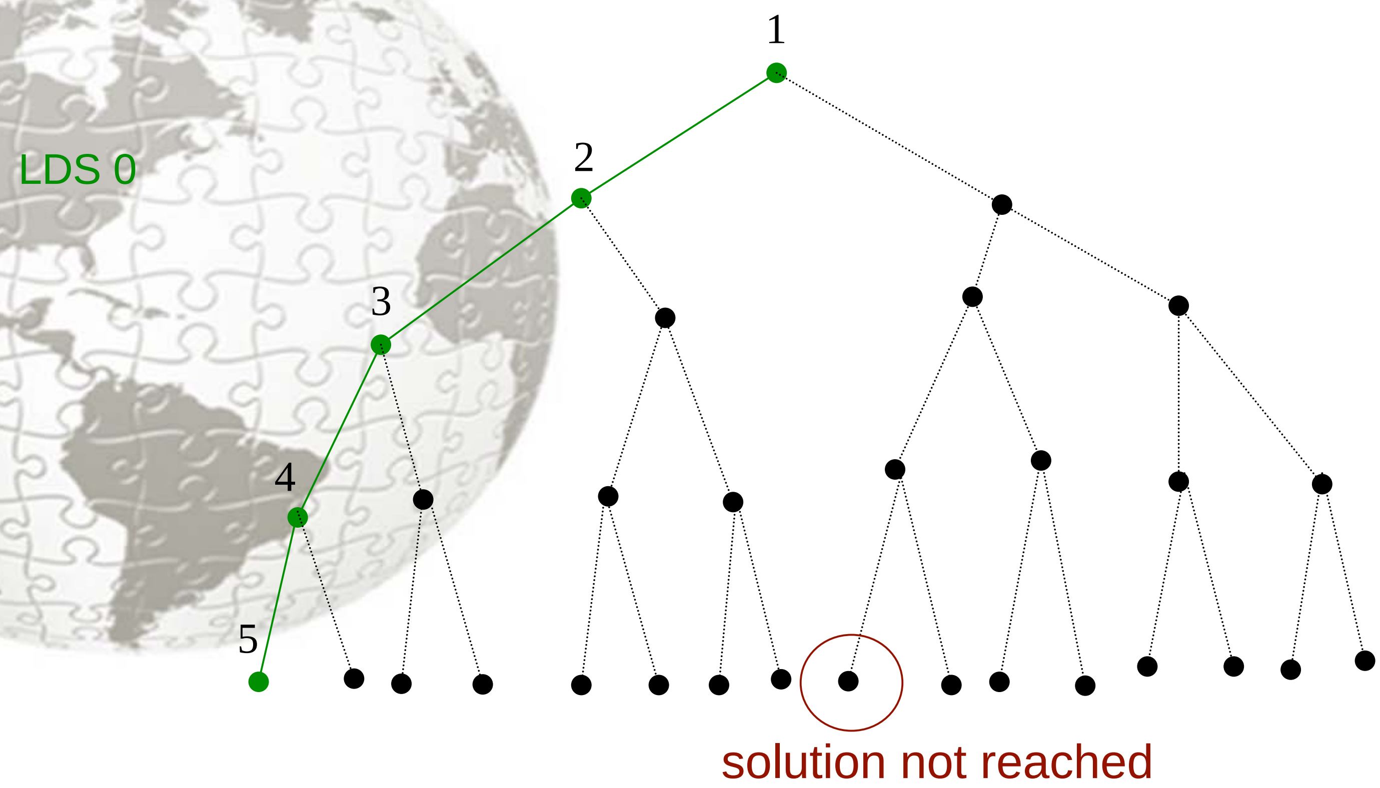
Depth First Search (DFS)

- Search principle
  - Example: binary tree



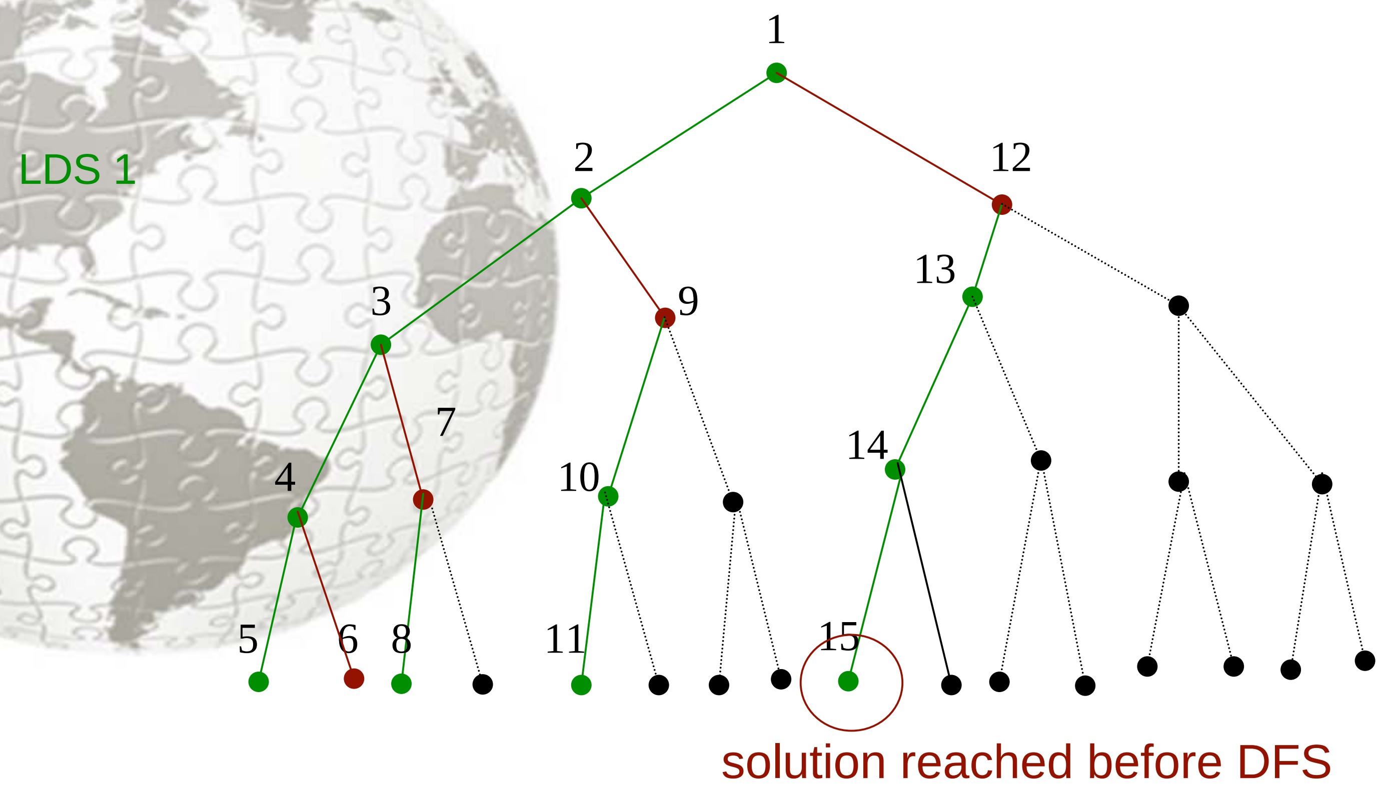
Limited Discrepancy Search (LDS)

- Search principle
  - Example: binary tree
  - LDS 0: The choices of the heuristic are satisfied



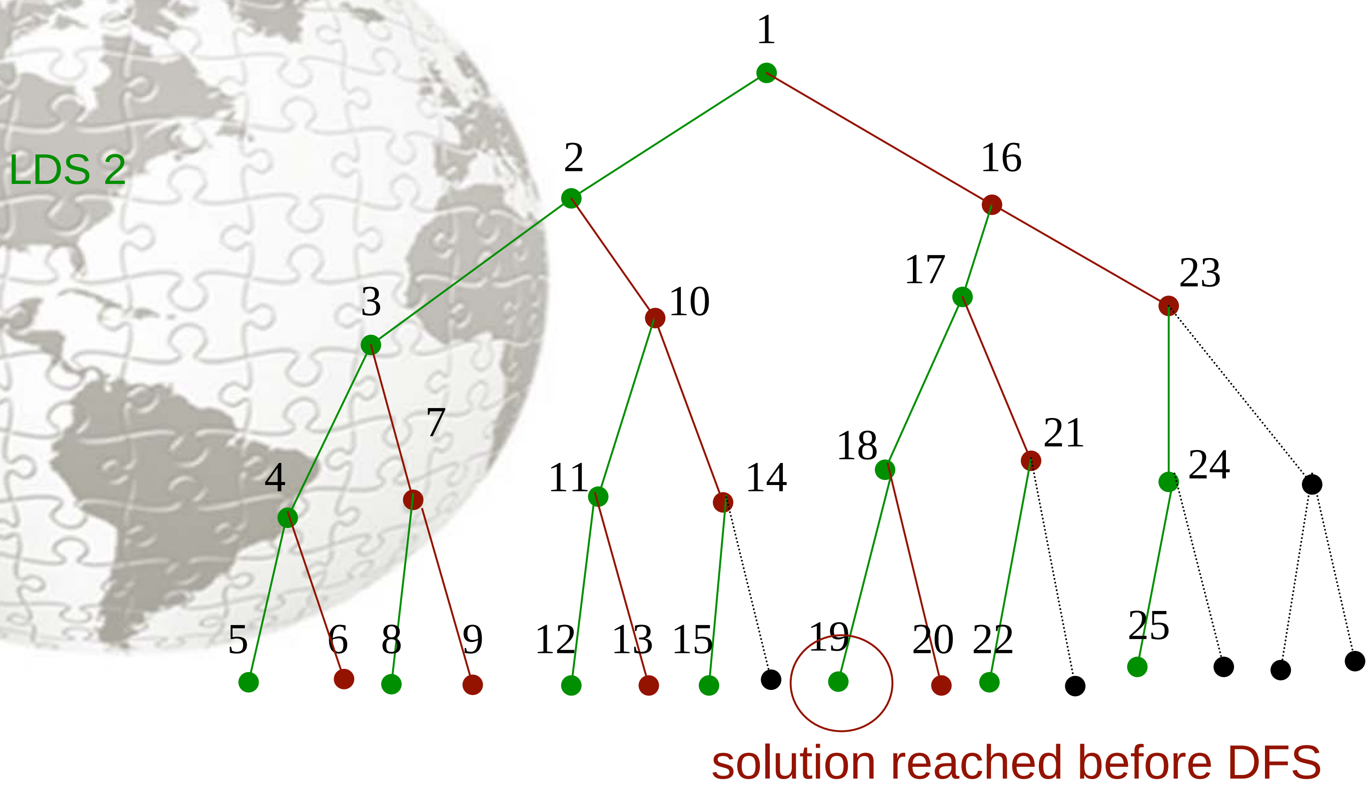
Limited Discrepancy Search (LDS)

- Search principle
  - Example: binary tree
  - LDS 1: All the paths differ of ONE decision from LDS 0



Limited Discrepancy Search (LDS)

- Search principle
  - Example: binary tree
  - LDS 2: All the paths differ of TWO decisions (for TWO variables) from LDS 0

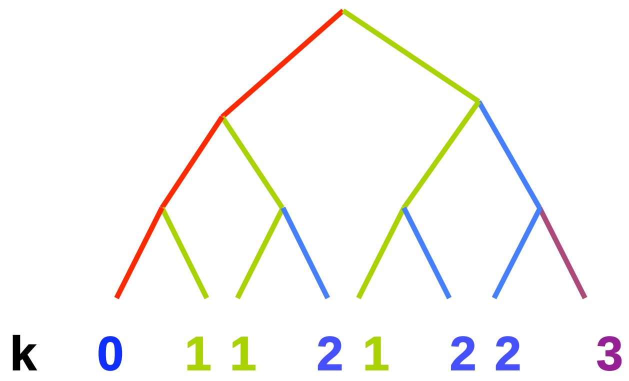


(Improved) Limited Discrepancy Search (I-LDS)

[Harvey & Ginsberg, 1995] and [Korf, 1996]

Algorithm

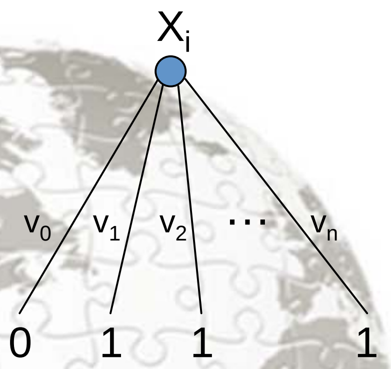
```
k ← 0
kmax ← N
I ← Initial_instantiation()
While no_solution() and (k ≤ kmax) do
    k ← k+1
      -- Generate leaves at discrepancy k from I
      -- Stop when a solution is found
     I ← compute_Leaves (I, k)
End while
```



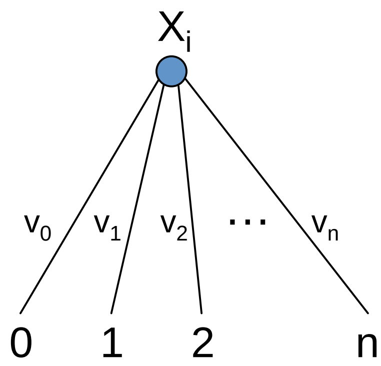
Universidad de los Andes

Discrepancy-based search methods (2)

- Non-binary trees: 2 ways for counting discrepancies



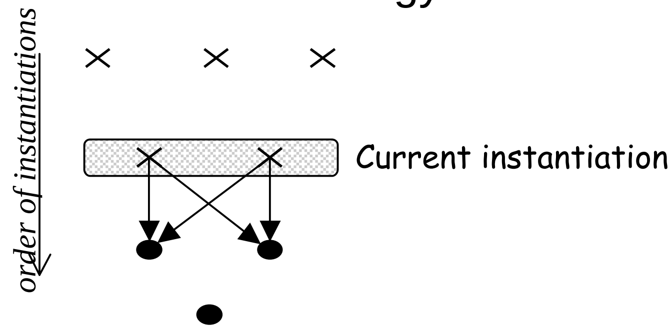
binary counting



non-binary counting

- The binary counting is adopted in our search strategy

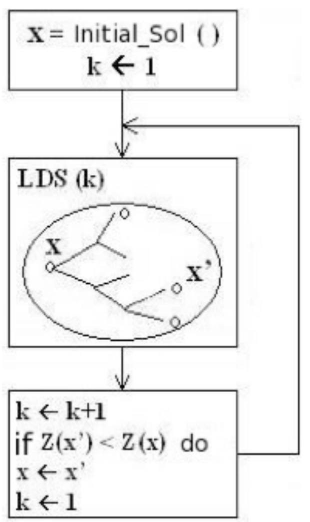
- Local propagation by *Forward-Checking*



Prospective scheme

Discrepancy-based search methods (3)

- Depth-bounded Discrepancy Search - DDS [Walsh, 1997]
  - → *Satisfaction*
  - To correct "early mistakes" (the most important)
  - Principle: discrepancies on top of the search tree (given depth)
  - Stop: a solution is found
- Climbing Discrepancy Search - CDS [Milano & Roli, 2005]
  - To improve current solution → *Optimization*
  - Principle:
    - `Initial solution (Reference)`
    - `Apply LDS principle to explore the neighborhood from this reference`
    - `Reference ← Improved_Solution`
    - `Restart with #discrepancy ← 0`
  - Stop: no more improvement, limit on time or #iterations reached
  - CDS is close to VNS [Hansen & Mladenovic, 2001]



Universidad de los Andes

Proposed method: CDDS (1)

- To combine 2 discrepancy-based methods
  - Climbing DS (neighborhood search)
  - Depth-bounded DS (neighborhood restricted at the top of the tree)
  - → *Climbing Depth-bounded Discrepancy Search (CDDS)*

- Optimization method: approximate solutions
  - Criterion = *makespan* minimization
  - A solution = UB

$$\boxed{\min C_{\max}}$$

- LBs to fathom nodes in the search tree

- Example: HFS

$$\mathrm{LB_l(i)} = \mathrm{ct_{il}} + \sum_{j=l+1}^{k} p_{ij} \quad \text{(where } \mathrm{ct_{il}} \text{ is the completion time of } O_{il}\text{)}$$

$$\mathrm{LBM[Sch^{(s)}(Y)]} = \begin{cases} \mathrm{ACT[Sch^{(s)}(Y)]} + \min\limits_{i \in J-Y} \{ \sum\limits_{s'=s+1}^{L} p_i^{(s')} \} & \text{if } \mathrm{ACT[Sch^{(s)}(Y)]} > \mathrm{MCT[Sch^{(s)}(Y)]} \\ \mathrm{MCT[Sch^{(s)}(Y)]} + \min\limits_{i \in Y} \{ \sum\limits_{s'=s+1}^{L} p_i^{(s')} \} & \text{if } \mathrm{ACT[Sch^{(s)}(Y)]} < \mathrm{MCT[Sch^{(s)}(Y)]} \\ \mathrm{ACT[Sch^{(s)}(Y)]} + \max\{ \min\limits_{i \in J-Y} \sum\limits_{s'=s+1}^{L} p_i^{(s')}, \min\limits_{i \in Y} \sum\limits_{s'=s+1}^{L} p_i^{(s')} \} & \text{if } \mathrm{ACT[Sch^{(s)}(Y)]} = \mathrm{MCT[Sch^{(s)}(Y)]} \end{cases} \quad (7)$$

Universidad de los Andes

Proposed method: CDDS (2)

- Exploration strategy
  - Instantiation heuristics
    1. Job selection: $X_i \in \{O_{11}, O_{12}, \ldots, O_{1s_1}, O_{21}, \ldots, O_{n1}, \ldots, O_{ns_n}\}$
       11. Earliest Start Time (EST)
       12. SPT or EDD or LDJ (job of longest duration)
    2. Machine selection (allocation): $A_i \in \{M_1, \ldots, M_m\}$
       Earliest Completion Time (ECT)
  - Propagation = Forward-Checking over:
    - Start time of subsequent operations
    - Availability date of selected machine
  - Discrepancies
    - On job selection for HFS
    - On both types of variables (job and machine selection) for FJS

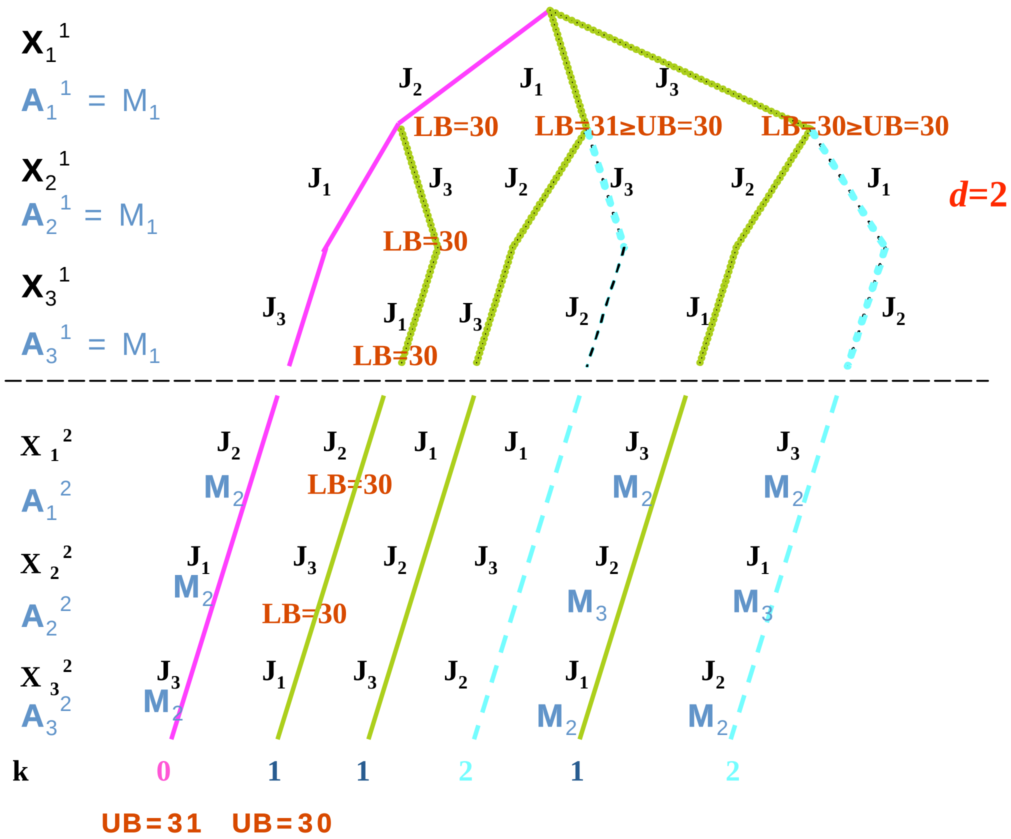
HFS: Example

Processing times of a 3×2 Hybrid Flow Shop

Jobs	*Stage 1*		*Stage 2*	
1	O_{11}	8	O_{12}	7
2	O_{21}	7	O_{22}	8
3	O_{31}	8	O_{32}	8

$M^{(1)} = 1;\ M^{(2)} = 2$

EST-SPT (1st stage):
J_2 ; J_1 ; J_3



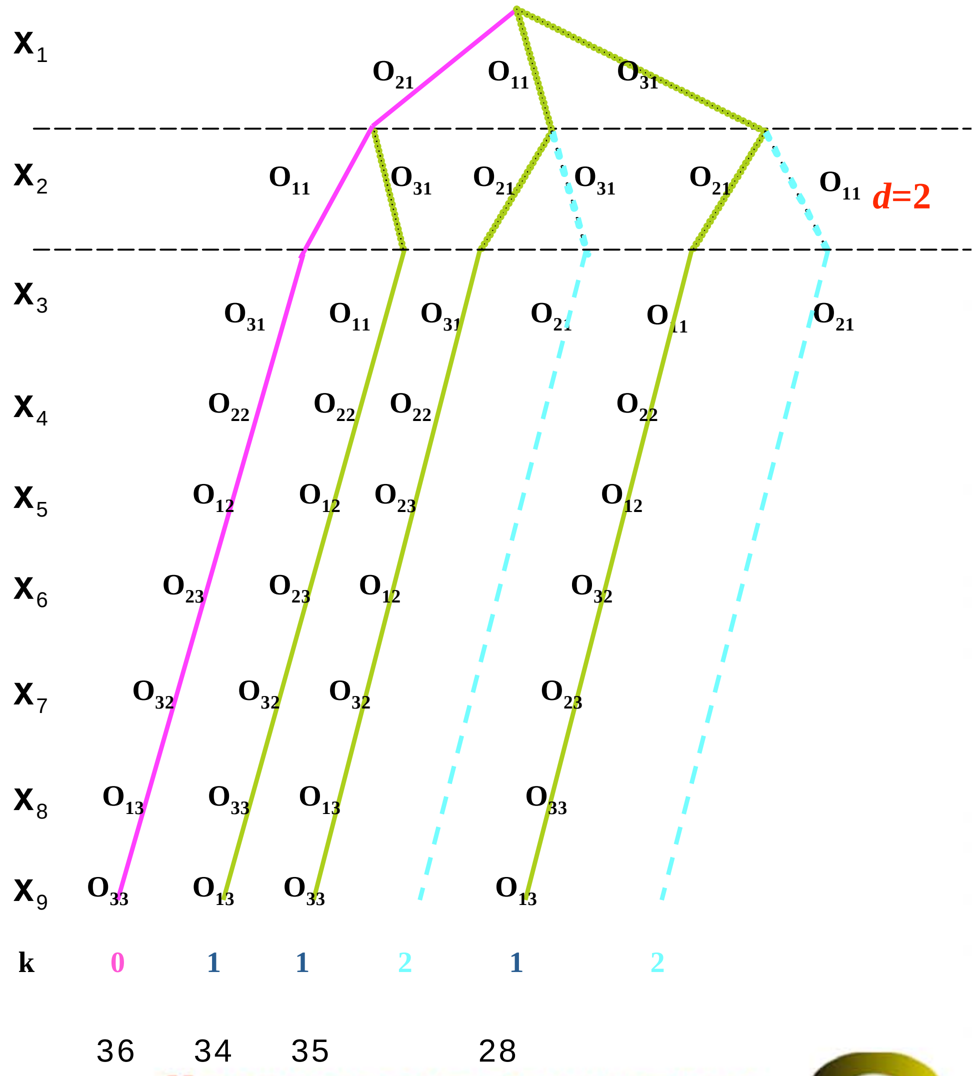
Universidad de los Andes

FJS: Example of discrepancy on job selection variable

Processing times of a 3×3 Flexible Job Shop

	M_1	M_2	M_3
O_{11}	--	8	--
O_{12}	--	8	--
O_{13}	--	6	--
O_{21}	11	2	--
O_{22}	--	3	10
O_{23}	11	--	10
O_{31}	12	--	12
O_{32}	4	--	12
O_{33}	12	--	--

EST-SPT: O_{21} ; O_{11} ; O_{31}



HFS: Experiments

- Instances:
  - Néron & Carlier
    - 52 easy problems
    - 24 hard problems
- Comparison:
  - LDJ is the best rule
  - B&B of [Néron & Carlier, 2000]
  - AIS
- Stop:
  - limit on CPU time=30 sec.

Relative performance of methods

Method	***easy***	***hard***	***all***
	% deviation / LB		
B&B	*2.21*	*6.88*	*3.68*
DDS	*1.42*	*8.01*	*3.58*
CDDS	*1.1*	*5.0*	*2.32*
AIS	***1.01***	***3.12***	***1.68***
CDDS^L	***0.96***	***3.06***	***1.62***

$$\% \text{ deviation} = \frac{C\max\_best - LowerBound}{LowerBound} \times 100$$

Universidad de los Andes

2 stage-HFS: Experiments

- Instances:
  - Three sets generated in a similar way as [Lee & Vairaktarakis, 1994]
    - Set A: $S_1[1-20]$; $S_2[1-40]$
    - Set B: $S_1[1-40]$; $S_2[1-20]$
    - Set C: $S_1[1-40]$; $S_2[1-40]$

    n={10,20,30,40,50,100,150} : 1680 instances.
- Comparison:
  - LBs [Haouari *et al.*, 2006]
  - TS and LBs [Haouari & M'Hallah, 1997]
- Stop:
  - limit on CPU time=30 sec.

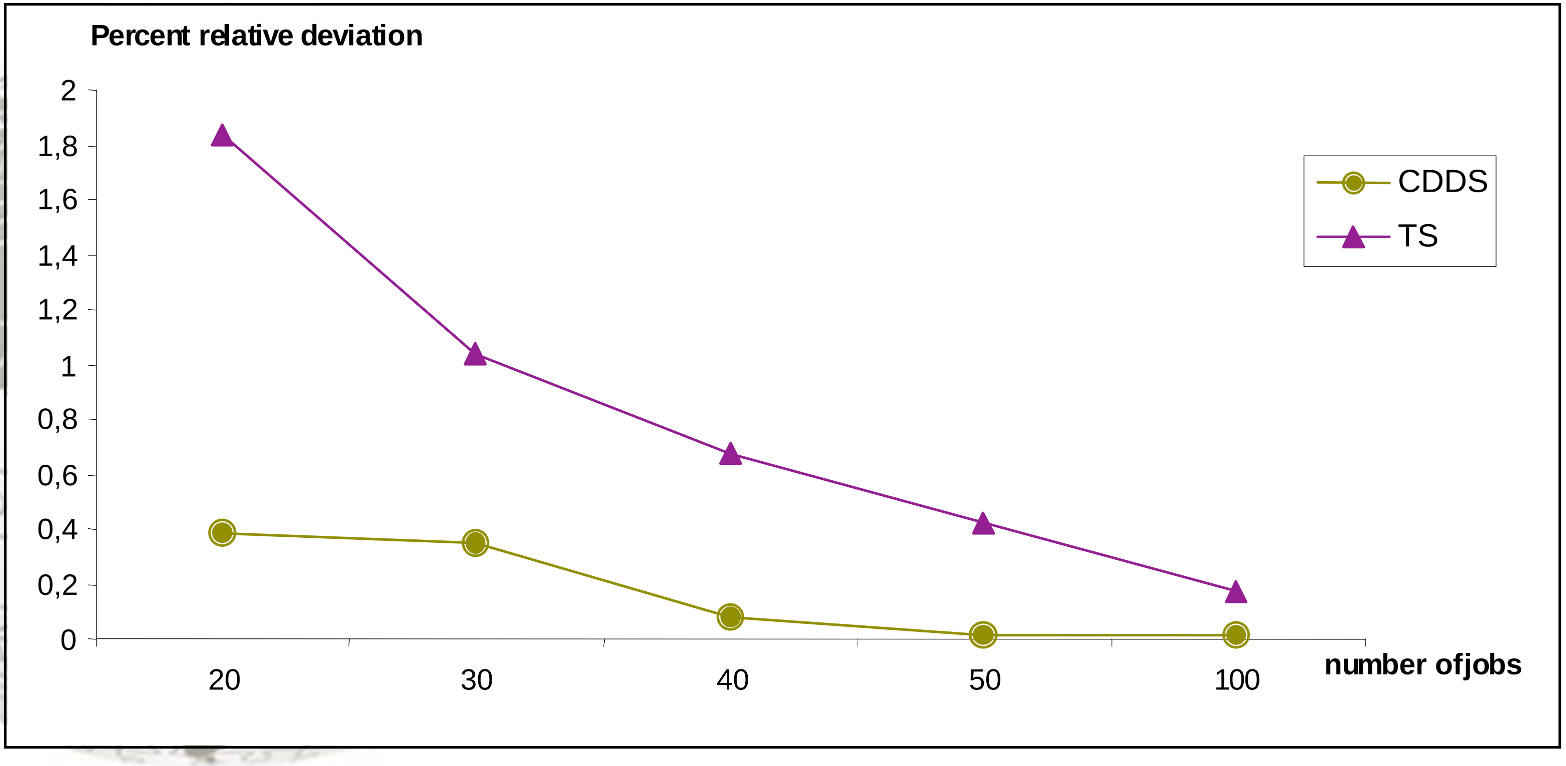
Relative performance of methods

Method	*Set A*	*Set B*	*Set C*
	% deviation / LBs_{2006}		
***CDDS**^{L}*	*0.82*	*0.33*	*0.34*
***CDDS**^{2}*	*0.17*	*0.13*	*0.22*
	% deviation / LBs_{1997}		
TS	*0.63*	*0.97*	*0.86*
***CDDS**^{2}*	*0.16*	*0.12*	*0.26*

$$\% \text{ deviation} = \frac{C\max\_best - LowerBound}{LowerBound} \times 100$$

Universidad de los Andes

2 stage-HFS: Experiments



Universidad de los Andes

FJS: Experiments

- Instances:
  - Brandimarte's benchmarks
  - 10 problems
  - n=[10 – 20]; m=[4 – 15]; n_i=[5 – 15]
- Comparison:
  - EDD is the best rule
  - Brandimarte's LBs
  - TS of [Mastrolilli & Gambardella, 2000] *(M.G.)*
- Stop: limit on CPU time=30 sec.

instances	*n*	*m*	*LB*	*M.G.*	*CDDS*	*%dev*	*CPU(M.G.)*	*CPU(CDDS)*
Mk01	10	6	36	**40**	**40**	0.0	0.01	0.1
Mk02	10	6	24	**26**	**26**	0.0	0.73	0.2
Mk03	15	8	204	**204***	**204***	0.0	0.01	0.2
Mk04	15	8	48	**60**	**60**	0.0	0.08	0.03
Mk05	15	4	168	**173**	182	5.2	0.96	0.2
Mk06	10	15	33	**58**	60	3.4	3.26	0.1
Mk07	20	5	133	144	**139**	-3.5	8.91	0.3
Mk08	20	10	523	**523***	**523***	0.0	0.02	0.8
Mk09	20	10	299	**307**	**307**	0.0	0.15	0.4
Mk10	20	15	165	**198**	212	7.1	7.69	0.3
Average						1.2	2.18	**0.26**

Universidad de los Andes

FJS: Experiments

- Instances:
  - Hurink's benchmarks
  - 129 problems (43 JSP): EData
    RData
    Vdata
  - n=[6 – 30]; m=[5 – 15]

Degree of flexibility

- Comparison:
  - EDD is the best rule
  - [Pezella *et al.*, 2007]
  - Tabu + GA + LBs
- Stop:
  - limit on CPU time=30 sec.

- Mean relative error / best_LB):

Problems	*Tabu (%)*	*CDDS (%)*	*GA (%)*
EData	*2.2*	*5.3*	*6.0*
RData	*1.2*	*2.5*	*4.4*
VData	*0.1*	*0.6*	*2.0*

$$\%\ \text{deviation} = \frac{C\max\_best - LowerBound}{LowerBound} \times 100$$

Universidad de los Andes

FJS: Experiments

Deviation percentage over the best known lower bound

Data set	num	alt	CDDS (%)
Brandimarte	10	2.59	1.2
Hurink Edata	43	1.15	5.3
Hurink Rdata	43	2	2.5
Hurink Vdata	43	4.31	0.6

num: number of instances; alt: machine's number per job

Universidad de los Andes

Parallel machine scheduling

- Comparisons on $Pm|r_i,q_i|C_{max}$ problems [Néron *et al.*, 2008]
  (50 hard instances; n = 100, m = 10, p_i = [1 − 10])
- Stop: limit on CPU time=30 sec.

$CPU_{limit} = 30\ s$	*Best Solution*	*Best Sol. Strict*	*CPU(s)*
$LDS^{TW}_{z=1}$	1	0	29.64
$LDS^{CHR}_{z=2}$	7	0	28.40
$BS^{TW}_{\omega=3}$	25	3	20.37
$BS^{CHR}_{\omega=4}$	22	0	28.40
CDS	35	6	30 (8.03)
HD-CDDS	38	9	30 (7.02)

Conclusions

- Novel method to solve Flexible Shop Problems:
  - CDDS: Climbing Depth-bounded Discrepancy Search
    - Hybrid Flow Shop
      → Excellent results - [Ben Hmida *et al.*, 2007])
    - Flexible Job Shop (results to confirm)
    - Parallel machine (with precedence constraints and setup times, L_{max}, ΣC_i
      → Excellent results - *[Comp. & Op. Res., under review]*)

Universidad de los Andes

Further works

- Hope of even better results on FJS with the introduction of adapted lower bounds

- *Backjumping heuristic* on promising choice points for making discrepancies
  1. Permutation of two adjacent critical operations carried out by the same resource (discrepancy on selection variable) [van Laarhoven et al., 1992]
  2. Replacement of a critical operation on another resource (discrepancy on allocation variable but restricted to critical operations)

- Extension to Multimode Resource-Constrained Project Scheduling Problems (MRCPSPs)

- Multicriteria FJS (ΣC_i ; $L_{\max}$ – [Vilcot & Billaut, 2007])