Scheduling under energy constraints

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Energy-aware scheduling

Computer systems

- Energy consumption management: critical issue in computer systems / networks / embbeded systems.
- Many (online) algorithmic problems raised [Irani and Pruhs, 2005].

Production scheduling

- Critical issue in process industries [Jovan, 2002].
- Growing interest in other areas.

Need of new models and methods to integrate energy management in standard production scheduling.

Constraint based scheduling [Baptiste et al., 1999]

Constraint programming and global constraints

- Clear distinction between modeling (through constraints) and solving (through tree search).
- Global constraints: both a declarative and an operational role.

Constraint programming for scheduling

- Decision variables : activity start times and resource allocation
- Global resource constraints : declare resource usage of activities and provide specific constraint propagation algorithms for feasibility tests and time-bound adjustments.

Define global constraints and search schemes for scheduling with energy resources.

The energy scheduling problem (EnSP)

Data

- a resource of availability B
- a set of non-preemptive activities $A = \{1, 2, \ldots, n\}$.
  - minimum power supply $b_i^{\min}$,
  - maximum power supply $b_i^{\max}$,
  - required energy W_i,
  - release date r_i,
  - deadline d_i
- a discrete set of time points $T = \{t_1, t_1 + 1, \ldots, t_2 - 1, t_2\}$ with $t_1 = \min_{i \in A} r_i$ and $t_2 = \max_{i \in A} d_i$.

The energy scheduling problem (EnSP)

Problem Statement

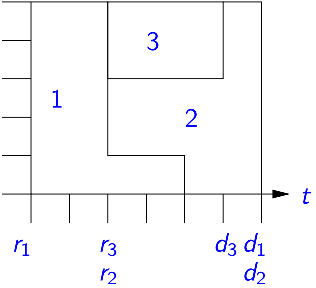
Find a start time $st_i \in T$, $i \in A$, a finishing time $ft_i \in T$, $i \in A$ and a resource consumption b_{it}, $i \in A$, $t \in T$.

- $st_i \geq r_i$, $i \in A$ (*release date constraints*)
- $ft_i \leq d_i$, $i \in A$, (*deadline constraints*)
- $b_i^{min} \leq b_{it} \leq b_i^{max}$ for $t \in \{st_i, ft_i - 1\}$ and $b_{it} = 0$ otherwise. (*minimal and maximal power supply constraints*)
- $W_i \leq \sum_{t=st_i}^{ft_i-1} b_{it}$ (*required energy constraints*)
- $\sum_{i \in A} b_{it} \leq B$, for $t \in T$ (*resource constraints*)

EnSP example

i	r_i	d_i	W_i	$b_i^{\min}$	$b_i^{\max}$
1	0	6	12	1	5
2	2	6	12	2	5
3	2	5	6	2	2

$B = 5$



Industrial application example

Pipe Manufacturing

- Problem issued from a Montreal company studied in [Trépanier et al., 2005, Haït et al., 2007]
- Metal is melted in induction furnaces.
- Melting operation has a variable duration that depends on the power given to the furnace.
- global allocated power not to be overran.
- Power change occurs only at fixed intervals (15mn)
- Further complicating constraints (loading/unloading operators, limited number of furnaces)

Related work

- The Cumulative Scheduling problem (CuSP) [Erschler and Lopez, 1990] $b_i^{\max} = b_i^{\min}, \forall i$.
- The fully elastic case [Baptiste et al., 1999] $b_i^{\min} = 0$, $b_i^{\max} = B, \forall i$.
- The partially elastic case [Baptiste et al., 1999] (no minimal and maximal consumption)
- The trapezoidal case [Poder and Beldiceanu, 2008] (fixed number of trapezoids, no amount of energy required)
- The discrete time-resource tradeoff [Ranjbar et al., 2009] (rectangular shape)

Complexity of the EnSP

Theorem

The decision variant of the EnSP is NP-complete in the strong sense

Trivial reduction from the CuSP by setting $b_i^{\min} = b_i$, $b_i^{\max} = b_i$ and $W_i = b_i p_i$.

A Constraint programming approach

Global energy constraint propagation

- Feasibility tests
- Time-bound adjustements on variables st_i and ft_i.

Tree search Method

- Dominance rules
- Branching scheme based on b_{it} variables

Feasibility tests for the EnSP

Basic feasibility test

If, for an activity i, $b_i^{\max}.(d_i - r_i) < W_i$, the EnSP is unfeasible.

If this condition is not verified, Let $\underline{w}(i, t_1, t_2)$ denote the minimal energy consumtion of i in interval $[t_1, t_2]$.

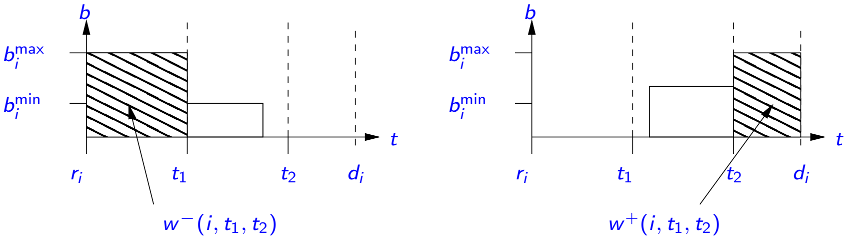
Interval feasibility test

If $\exists [t_1, t_2]$ verifying $\sum_{i \in A} \underline{w}(i, t_1, t_2) > B.(t_2 - t_1)$, the EnSP is unfeasible.

Maximum consumption outside the interval

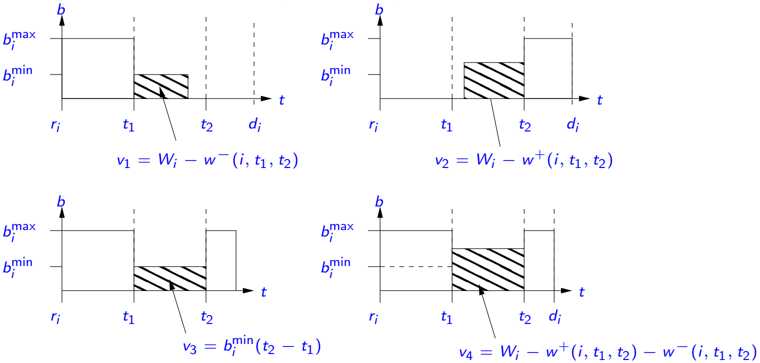
The minimum consumption of i in $[t_1, t_2]$ is attained either when i is left-shifted or right shifted. Let

- $w^-(i, t_1, t_2) = \min\{W_i, \max(0, b_i^{\max}(t_1 - r_i))\}$ the maximum energy consumed by i before t_1
- $w^+(i, t_1, t_2) = \min\{W_i, \max(0, b_i^{\max}(d_i - t_2))\}$ the maximum energy consumed by i after t_2



Minimum energy consumption computation

Consider the minimum $v = \min(v_1, v_2, v_3)$.



$\underline{w}(i, t_1, t_2) =$

- 0 if $v = 0$
- $\max(b_i^{min}, v, v_4)$ otherwise

Interval slack, left and right minimum consumptions

Interval slack of an activity

The maximum available energy (i.e. the slack) for processing i on $[t_1, t_2]$ is equal to $SL(i, t_1, t_2) = B.(t_2 - t_1) - \sum_{j \in A \setminus \{i\}} \underline{w}(j, t_1, t_2)$

Left and right minimum consumptions

- $w_L(i, t_1, t_2)$ the minimal energy consumption of i in $[t_1, t_2]$ when i is left shifted (i.e. $st_i = r_i$)
- $w_R(i, t_1, t_2)$ the minimal energy consumption of i in $[t_1, t_2]$ when i is right shifted (i.e. $ft_i = d_i$)

Time bound adjustments for the EnSP

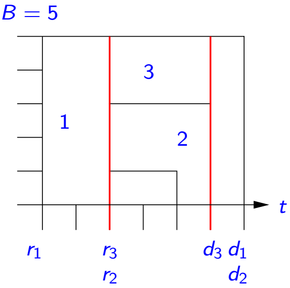
Release date adjustment

$\exists i, [t_1, t_2],\ w_L(i, t_1, t_2) > SL(i, t_1, t_2) \implies$
$r_i \leftarrow \max\{r_i, \lceil t_2 - SL(i, t_1, t_2)/b_i^{\min} \rceil\}$.

Deadline adjustment

$\exists i, [t_1, t_2],\ w_R(i, t_1, t_2) > SL(i, t_1, t_2) \implies$
$d_i \leftarrow \min\{d_i, \lfloor t_1 + SL(i, t_1, t_2)/b_i^{\min} \rfloor\}$.

Time bound adjustments: Illustration



Interval $[t_1, t_2] = [2, 5]$
$\underline{w}(1, t_1, t_2) = w_L(1, t_1, t_2) = 2$, $\underline{w}(2, t_1, t_2) = w_R(3, t_1, t_2) = 7$ and
$\underline{w}(3, t_1, t_2) = w_L(3, t_1, t_2) = w_R(3, t_1, t_2) = 6$
$w_R(1, t_1, t_2) = 7$ and
$SL(1, t_1, t_2) = 15 - \underline{w}(3, t_1, t_2) + \underline{w}(2, t_1, t_2) = 2 \implies d_1 \leftarrow t_1 + 2/1 = 4$

Constraint propagation algorithm

- Apply the feasibility tests and the time bound adjustment rules for each $[t_1, t_2] \in \{r_i | i \in A\} \times \{d_i | i \in A\}$.
- time complexity $O(n^3)$
- **This set of interval is not sufficient (known from the CuSP)**.

Dominance rules for the EnSP

- Active schedules are dominant
- Schedules for which, for any activity i, changes in the allocated power only occur on activity release dates, or completion times are dominant.

Branching scheme for the EnSP

At each node, associated to a time point t the activities are partitionned into the following sets

- *started* activities. st_i assigned at $t' \leq t$, but b_{it} not assigned
- *processed* activities. st_i assigned at $t' \leq t$, b_{it} assigned.
- *completed* activities. $ft_i \leq t$
- *available* activities. $r_i \leq t$, st_i not assigned.
- *unavailable* activities. $r_i > t$.
- *postponed* activities. $st_i > t$.

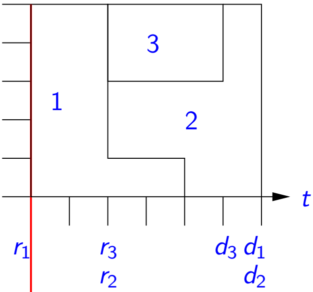
Branching scheme for the EnSP

(Extension of the “schedule or postpone” branching scheme)

- Select a *started* or *available* activity for being *processed* at t with b_{it} time units, trying first the maximal resource units available or *postpone* it.
- Prune node as soon as schedule is not semi-active anymore.
- if not enough available resource, set t to the smallest next release date or completion time or prune the node if the *started* set is not empty.

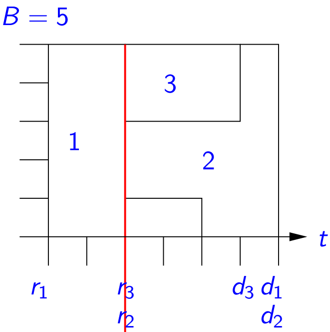
Branching scheme for the EnSP: illustration

$B = 5$



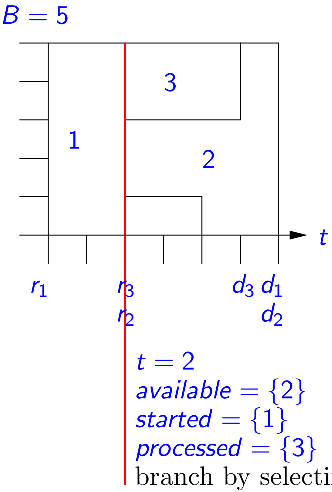
$t = 0$
$available = \{1\}$
$unavailable = \{2, 3\}$
branch by selecting 1 with $b_{it} = 5$

Branching scheme for the EnSP: illustration



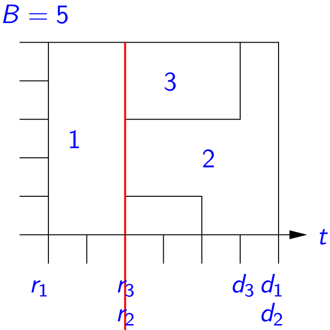
$t = 2$
$available = \{2, 3\}$
$started = \{1\}$
branch by selecting 3 with $b_{it} = 2$

Branching scheme for the EnSP: illustration



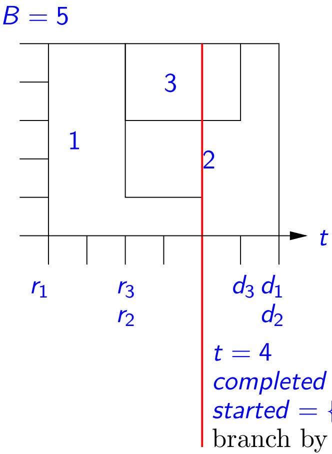
$t = 2$
$available = \{2\}$
$started = \{1\}$
$processed = \{3\}$
branch by selecting 2 with $b_{it} = 2$

Branching scheme for the EnSP: illustration



$t = 2$
$started = \{1\}$
$processed = \{2, 3\}$
branch by selecting 1 with $b_{it} = 1$

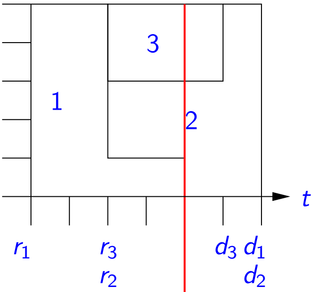
Branching scheme for the EnSP: illustration



$t = 4$
$completed = \{1\}$
$started = \{2, 3\}$
branch by selecting 3 with $b_{it} = 2$

Branching scheme for the EnSP: illustration

$B = 5$



$t = 4$

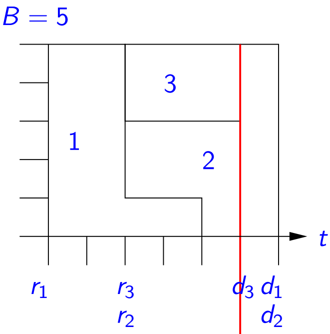
$completed = \{1\}$

$started = \{2\}$

$processed = \{3\}$

branch by selecting 2 with $b_{it} = 3$

Branching scheme for the EnSP: illustration



$t = 5$
$completed = \{1, 3\}$
$started = \{2\}$
branch by selecting 2 with $b_{it} = 5$

Preliminary computational results

Tree search with and without energy reasoning on 5 problem instances.

			results with energy reasoning			results without energy reasoning		
Inst.	n	B	Solution	CPU (s)	#Nodes	Solution	CPU (s)	#Nodes
1	25	10	Infeasible	0	1	NA	NA	$> 10^6$
2	25	10	Feasible	6	101970	Feasible	17	464933
3	25	10	Feasible	13	212382	Feasible	16	443491
4	25	10	Feasible	22	368264	Feasible	31	811607
5	30	10	Infeasible	27(2)	69350(69350)	Infeasible	4	173819

Conclusion and further work

- A New global constraint for scheduling with energy constraints
- An efficient constraint propagation algorithm

TODO list:

- Theoretical study of the relevant intervals
- More computational results: comparison with state-of-the-art constraint propagation algorithms (elastic CuSP...)
- Applications : electrical energy, manpower

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