Event-based MIP models for the resource constrained project scheduling problem

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The resource constrained project scheduling problem (1/3)

Data

- $A = \{1, \ldots, n\}$ set of activities
- p_i duration of activity $i \in A$
- $R = \{1, \ldots, m\}$ set of resources
- B_k number of available units of resource $k \in R$
- b_{ik} number of units of resource $k \in R$ required by activity $i \in A$
- $H = \{0, \ldots, T-1\}$ discrete set of time periods.
- E set of precedence constraints

A RCPSP example with $m = 2$ and $n = 10$

i	1	2	3	4	5	6	7	8	9	10
p_i	7	3	5	5	6	4	5	4	3	7
b_{1i}	0	2	3	3	2	1	1	1	1	3
b_{2i}	2	1	3	2	1	0	3	1	1	1
Successors	3	6,7	4,9	11	1	1	5,8	10	4	9

The resource constrained project scheduling problem (2/3)

Feasible schedule

- $S_i \in H$ start time of activity $i \in A$ in schedule S
- $\mathcal{A}(S,t) = \{i \in A | S_i \leq t \leq S_i + p_i - 1\}$ set of activities in progress at time $t \in H$ for schedule S.
- $\mathcal{S}$ set of feasible schedule where $S \in \mathcal{S}$ verifies $S_j \in H$, $S_j \geq S_i + p_i$, $\forall (i,j) \in E$ and $\sum_{i \in A(S,t)} b_{ik} \leq B_k$, $\forall k \in R$

RCPSP

- $\min_{S \in \mathcal{S}} max_{i \in A}(S_i + p_i)$

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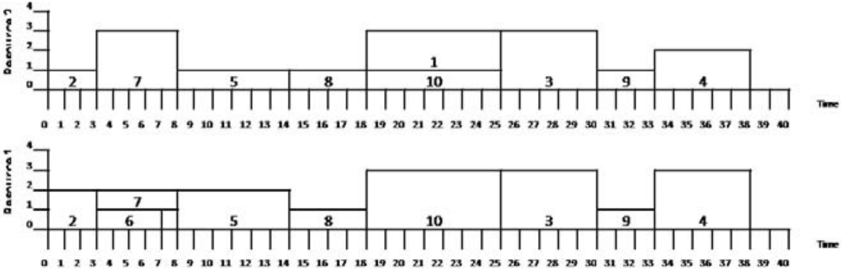
p_i	7	3	5	5	6	4	5	4	3	7
b_{1i}	0	2	3	3	2	1	1	1	1	3
b_{2i}	2	1	3	2	1	0	3	1	1	1
Successors	3	6,7	4,9	11	1	1	5,8	10	4	9

Feasible schedule



Overview of state-of-the-art methods

- Heuristics [Kolish and Hartmann 2006]
  MIP search can be used to solve subproblems in a large neighborhood framework [Palpant et al 2004]
- Lower bounds [Néron et al 2006]
  Best bounds combine constraint propagation and linear programming relaxations
- Exact methods [Demeulemeester and Herroelen 1997] [Sprecher 2000] [Laborie 2005]
  Best methods based on problem structure, MIP results not reported on the standard benchmark instances

What is the performance of today's MIP solvers for solving exactly the RCPSP (through compact formulations)?

Compact MIP formulations for the RCPSP 1/3

Basic Discrete time formulation (DT) [Pritsker *et al.* 1969]

$$\min_x \sum_{t \in H} t x_{n+1,t}$$

$$\sum_{t \in H} t x_{jt} \geq \sum_{t \in H} t x_{it} + p_i \qquad \forall (i,j) \in E$$

$$\sum_{i=1}^{n} b_{ik} \sum_{\tau = t - p_i + 1}^{t} x_{i\tau} \leq B_k \qquad \forall t \in H, \forall k \in R$$

$$\sum_{t \in H} x_{it} = 1 \qquad \forall i \in A \cup \{0, n+1\}$$

$$x_{it} \in \{0, 1\} \qquad \forall i \in A \cup \{0, n+1\}, \forall t \in H$$

O and $n+1$ dummy start and end activities. $(n+2)T$ **binary variables**

Disaggregated Discrete time formulation (DDT) [Christofides *et al.* 1987]

$$\min_x \sum_{t \in H} t x_{n+1,t}$$

$$\sum_{\tau=t}^{T} x_{i\tau} + \sum_{\tau=0}^{t+p_i-1} x_{j\tau} \leq 1, \qquad \forall t \in H, \forall (i,j) \in E$$

$$\sum_{i=1}^{n} b_{ik} \sum_{\tau=t-p_i+1}^{t} x_{i\tau} \leq B_k \qquad \forall t \in H, \forall k \in R$$

$$\sum_{t \in H} x_{it} = 1 \qquad \forall i \in A \cup \{0, n+1\}$$

$$x_{it} \in \{0,1\} \qquad \forall i \in A \cup \{0, n+1\}, \forall t \in H$$

$(n+2)T$ **binary variables**

Compact MIP formulations for the RCPSP 3/3

Flow-based continuous-time formulation (FCT) [Artigues *et al.* 2003]

$$\min_{S,f,x} S_{n+1}$$

$$S_j - S_i \geq -M + (p_i + M)x_{ij} \qquad \forall (i,j) \in (A \cup \{0, n+1\})^2$$

$$f_{ijk} \leq \min(b_{ik}, b_{jk})x_{ij} \qquad \forall (i,j) \in (A \cup \{0\} \times A \cup \{n+1\}), \forall k \in R$$

$$\sum_{j \in A \cup \{0,n+1\}} f_{ijk} = \tilde{b}_{ik} \qquad \forall i \in A \cup \{0, n+1\}, \forall k \in R$$

$$\sum_{i \in A \cup \{0,n+1\}} f_{ijk} = \tilde{b}_{jk} \qquad \forall i \in A \cup \{0, n+1\}, \forall k \in R$$

$$x_{ij} = 1 \qquad \forall (i,j) \in E$$

$$f_{ijk} \geq 0 \qquad \forall (i,j) \in (A \cup \{0, n+1\})^2, \forall k \in R$$

$$f_{n+1,0k} = B_k \qquad \forall k \in R$$

$$S_i \geq 0 \qquad \forall i \in A \cup \{0, n+1\}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in (A \cup \{0, n+1\})^2$$

$(n+2)^2$ binary variables, $m(n+2)^2 + n + 2$ continuous variables

Benchmark instances and instance indicators (1/2)

Benchmark instances

- **KSD30**, 30 activities, 4 resources
- **BL**: Baptiste and Le Pape 2000, 20 and 25 activities, 3 resources
- **Pack**: Carlier and Néron 2001, 17 to 35 activities, 3 resources
- **KSD15_d**: keep 15 first activities of KSD30 and multiply some p_i by 15.
- **Pack_d**: Pack instances where some p_i are miltiplied by 50

Instance indicators

- **OS**: *Order strength*
- **NC**: *Network complexity*
- **RF**: *Resource factor*
- **RS**: *Resource Strength*
- **DR** *Disjunction Ratio*
- **PR** *Processing time Range*

Benchmark instances and instance indicators (2/2)

	KSD30	BL	Pack	KSD15_d	Pack_d
$\|V\|$	32	22 - 27	17 - 35	17	17 - 35
$\|R\|$	4	3	2 - 5	4	2 - 5
T	34 - 130	14 - 34	23 - 139	187 - 999	644 - 3694
OS	0.34 - 0.69	0.25 - 0.45	0.13 - 0.48	0.34 - 0.64	0.13 - 0.48
NC	1.5 - 2.13	1.45 - 2	1.5 - 1.72	1.18 - 1.82	1.50 - 1.72
RF	0.25 - 1.0	0.5 - 0.77	1 - 1	0.25 - 1	1
RS	0.14 - 1	0.16 - 0.55	0.08 - 0.53	0.18 - 1	0.08 - 0.48
DR	0.36 - 0.9	0.25 - 0.45	0.19 - 0.94	0.35 - 0.90	0.19 - 0.94
PR	10	5	19	250	1138

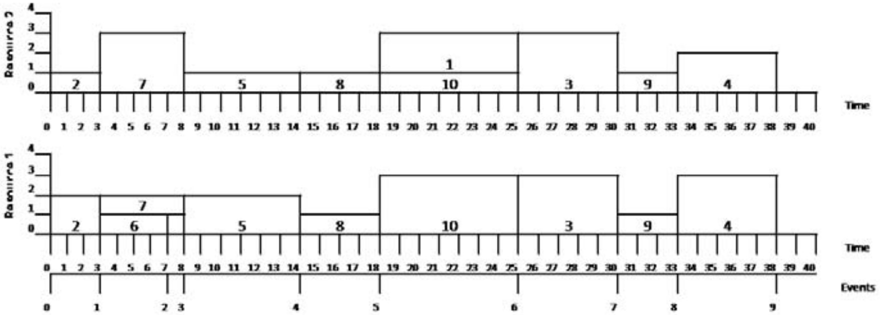
Experimental comparison of MIP formulations

- known result : LP relaxation DDT $\geq$ DT $\geq$ FCT
- Comparison of exact solving through branch and bound (CPLEX 11, 500s on a XEON 5110 biprocessor Dell PC clocked at 1.6Ghz with 4GB RAM) (gap from best known solutions)

Instances	Formulations	Opt. Sol.		Non-opt. Sol.		Total Sol.	No sol.
		%	Time	%	% Gap	%	%
KSD30	DT	78	12.76	8	6	86	14
	DDT	82	10.45	9	5	91	9
	FCT	52	33.81	4	2	56	44
Pack	DT	64	37.32	9	2	73	27
	DDT	73	61.09	22	127	95	5
	FCT	0	0.00	2	13	2	98
BL	DT	100	37.93	0	0	100	0
	DDT	100	13.68	0	0	100	0
	FCT	0	0.00	0	0	0	100
KSD15_d	DT	55	6.34	1	0	56	44
	DDT	5	1.65	0	0	5	95
	FCT	95	7.87	4	0	99	1
Pack_d	DT	0	0.00	0	0	0	100
	DDT	0	0.00	0	0	0	100
	FCT	4	7.58	0	0	4	96

Can we solve “highly cumulative” instances with a high processing time range with MIP a solver ?

Idea : reducing the number of variables through the concept of event (cf machine scheduling [Lasserre and Queyranne 1992] [Dauzère-Pérès and Lasserre 1995] [Pinto and Grossmann 1995])



Can we adapt these formulations to discrete resources ?

New Event-Based MIP Formulations 2/5

How many events ($\mathcal{E}$ set of events) ?

$\forall i \in A$ either $S_i = 0$ or $\exists j \in A$, $S_i = S_j + p_j \implies |\mathcal{E}| \leq n + 1$

Start/End Event-based formulation (SEE)

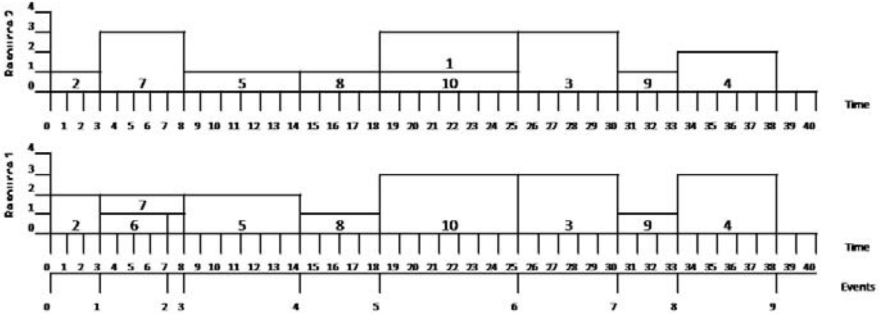
- Variable $x_{ie} \in \{0, 1\}$: activity i starts at event e.
- Variable $y_{ie} \in \{0, 1\}$: activity i ends at event e.
- t_e time of event e
- $2n^2 + 2n$ binary variables, $(n + 1)$ continuous variables

On/Off Event-based formulation (OOE)

- Variable $z_{ie} \in \{0, 1\}$: z_{ie} is set to 1 if activity i starts at event e or if it still being processed immediately after event e
- activity i starts at event e or if it still being processed immediately after event e.
- n^2 binary variables, $(n + 1)$ continuous variables

New Event-Based MIP Formulations 3/5

Example



e	0	1	2	3
x_{6e}	0	1	0	0
y_{6e}	0	0	1	0
z_{6e}	0	1	0	0
x_{7e}	0	1	0	0
y_{7e}	0	0	0	1
z_{7e}	0	1	1	0

t	2	3	4	5	6	7	8
x_{6t}	0	1	0	0	0	0	0
x_{7t}	0	1	0	0	0	0	0

Start/End Event-based formulation (SEE)

$$\min t_n$$

$$t_0 = 0$$

$$t_f \geq t_e + p_i x_{ie} - p_i(1 - y_{if}) \quad \forall (e,f) \in \mathcal{E}^2, f > e, \forall i \in A$$

$$t_{e+1} \geq t_e \quad \forall e \in \mathcal{E}, e < n$$

$$\sum_{e \in \mathcal{E}} x_{ie} = 1, \quad \sum_{e \in \mathcal{E}} y_{ie} = 1 \quad \forall i \in A$$

$$\sum_{e'=e}^{n} y_{ie'} + \sum_{e'=0}^{e-1} x_{je'} \leq 1 \quad \forall (i,j) \in E, \forall e \in \mathcal{E}$$

$$r_{0k} = \sum_{i \in A} b_{ik} x_{i0} \quad \forall k \in K$$

$$r_{ek} = r_{(e-1)k} + \sum_{i \in A} b_{ik} x_{ie} - \sum_{i \in A} b_{ik} y_{ie} \quad \forall e \in \mathcal{E}, e \geq 1, k \in R$$

$$r_{ek} \leq B_k \quad \forall e \in \mathcal{E}, k \in R$$

$$x_{ie} \in \{0,1\}, y_{ie} \in \{0,1\} \quad \forall i \in A \cup \{0, n+1\}, \forall e \in \mathcal{E}$$

$$t_e \geq 0, r_{ek} \geq 0 \quad \forall e \in \mathcal{E}, k \in R.$$

On/Off Event-based formulation (OOE)

$$\min C_{\max}$$

$$C_{\max} \geq t_e + (z_{ie} - z_{i(e-1)})p_i \qquad \forall e \in \mathcal{E}, \forall i \in A$$

$$t_0 = 0, \, t_{e+1} \geq t_e \qquad \forall e \neq n-1 \in \mathcal{E}$$

$$t_f \geq t_e + ((z_{i-e} - z_{i(e-1)}) - (z_{if} - z_{i(f-1)}) - 1)p_i \qquad \forall (e,f,i) \in \mathcal{E}^2 \times A, f > e \neq 0$$

$$\sum_{e'=0}^{e-1} z_{ie'} \geq e(1 - (z_{ie} - z_{i(e-1)})), \; \sum_{e'=e}^{n-1} z_{ie'} \geq e(1 + (z_{ie} - z_{i(e-1)})) \qquad \forall e \neq 0 \in \mathcal{E}$$

$$\sum_{e \in \mathcal{E}} z_{ie} \geq 1 \qquad \forall i \in A$$

$$z_{ie} + \sum_{e'=0}^{e} z_{je'} \leq 1 + (1 - z_{ie})e \qquad \forall e \in \mathcal{E}, \forall (i,j) \in E$$

$$\sum_{i=0}^{n-1} b_{ik} z_{ie} \leq B_k \qquad \forall e \in \mathcal{E}, \forall k \in R$$

$$t_e \geq 0 \qquad \forall e \in \mathcal{E}$$

$$z_{ie} \in \{0,1\} \qquad \forall i \in A, \forall e \in \mathcal{E}$$

Computational evaluation of the new formulations

- poor LP relaxations
- Comparison of exact solving through branch and bound (CPLEX 11, 500s on a XEON 5110 biprocessor Dell PC clocked at 1.6Ghz with 4GB RAM)

Instances	Formulations	Opt. Sol.		Non-opt. Sol.		Total Sol.	No sol.
		%	Time	%	% Gap	%	%
KSD30	DDT	82	10.45	9	5	91	9
	SEE	3	123.62	0	4	3	97
	OOE	24	112.62	9	5	33	67
Pack	DDT	73	61.09	22	127	95	5
	SEE	0	0.00	0	0	0	100
	OOE	27	20.63	18	127	45	55
BL	DDT	100	13.68	0	0	100	0
	SEE	0	0.00	8	13	8	92
	OOE	0	0.00	49	0	49	51
KSD15_d	FCT	95	7.87	4	0	99	1
	SEE	76	10.95	18	1	94	6
	OOE	82	2.96	18	0	100	0
Pack_d	FCT	4	7.58	0	0	4	96
	SEE	4	215.08	0	0	4	96
	OOE	18	75.58	42	0	60	40

Concluding remarks and further research

- No dominant MIP formulation in terms of direct exact solving through CPLEX
- Guidelines for the choice of the accurate MIP formulation
  - Small horizon and integer durations : use DDT
  - Large horizon or non integer durations
    - "Disjunctive" problems : use FCT
    - "Cumulative" problems : use OOE
- Further research : find a dominant formulation, improve LP relaxation, column generation, play with the number of events...

Research report:

O. Koné, C. Artigues, P. Lopez, and M. Mongeau. Event-based MILP models for resource-constrained project scheduling problems. **LAAS report 09102**, Université de Toulouse, LAAS-CNRS, Toulouse, France, March 2009. (www.laas.fr/~artigues)