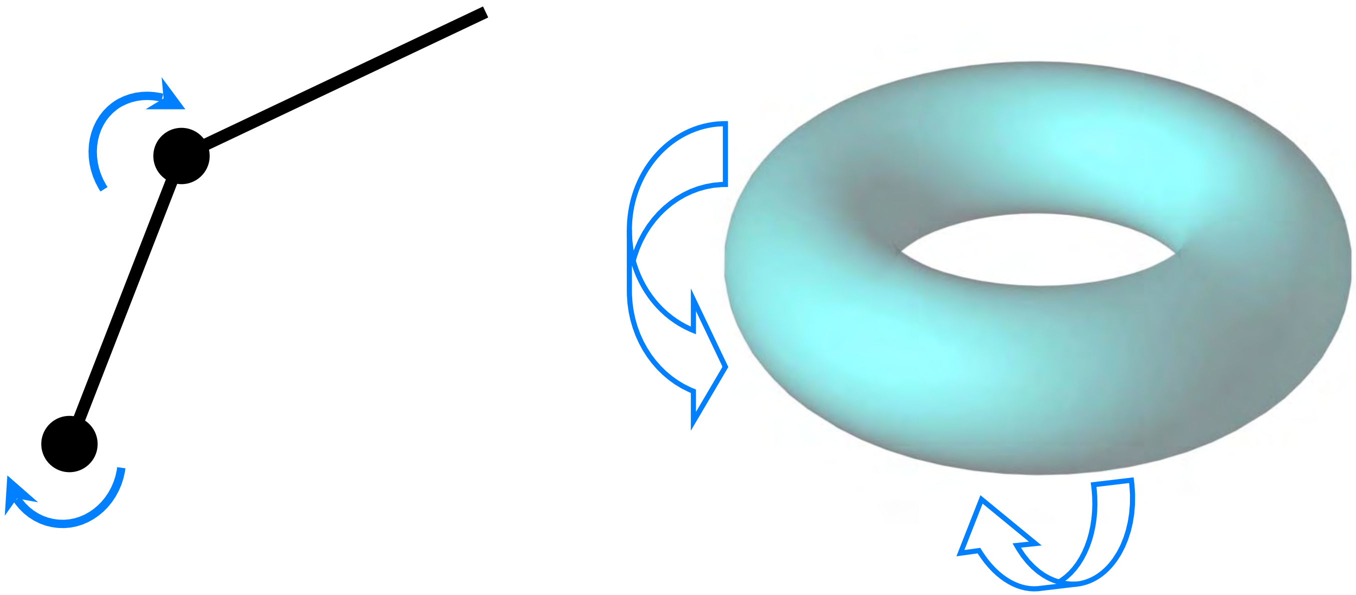
Mathematical foundations of robot motion

Jean-Paul Laumond

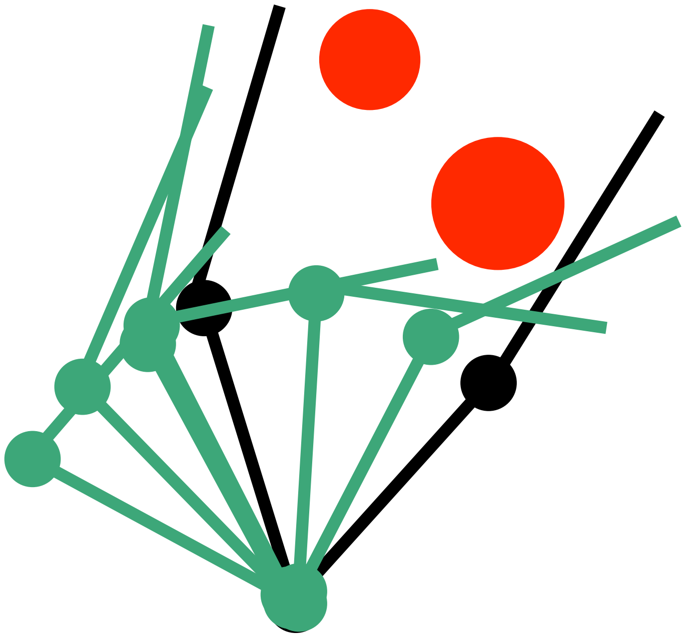
« Une aventure scientifique **et** humaine »

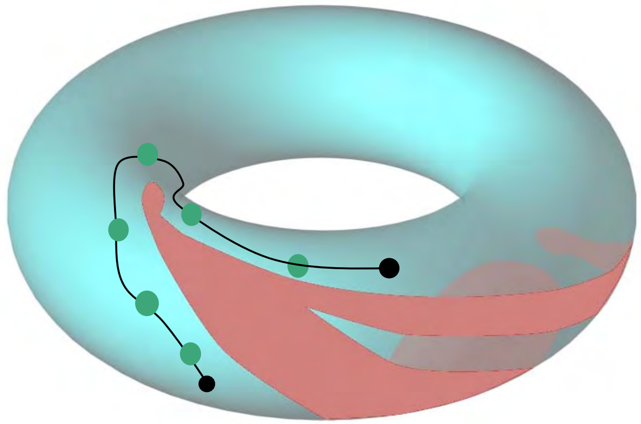
A sea of faces

Motion : a continuous function from time to space



Motion : a continuous function from time to space





- Any admissible motion for the **3D mechanical system** appears a collision-free path for a **point** in the CSpace

- Translating the ***continuous*** problem into a ***combinatorial*** one

- Capturing the ***topology*** of *CSfree* with ***graphs***

The 80's

- The « piano mover » problem is decidable

- Proof: Elementary algebra decidability

The 80's


- The « piano mover » problem is decidable

- Doubly exponential constructive proof:
  Cylindrical algebraic decomposition

- The 80's


- The « piano mover » problem is decidable

- Single exponential constructive proof: Retraction

The 80's

- The « piano mover » problem is decidable
- NP-hardness

The 80's

- Existence of motion in contact
- Proof: Algebraic topology

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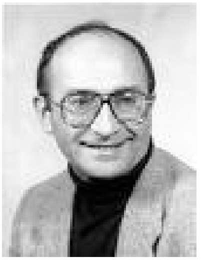
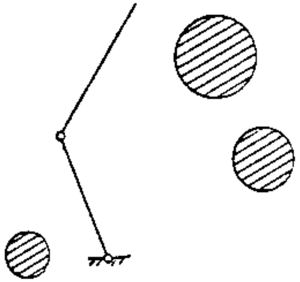
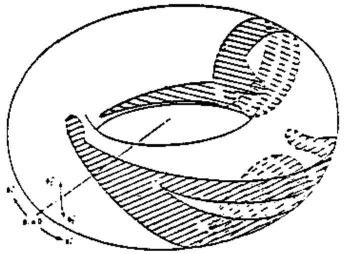
- Existence of motion in contact
- Proof: Algebraic topology

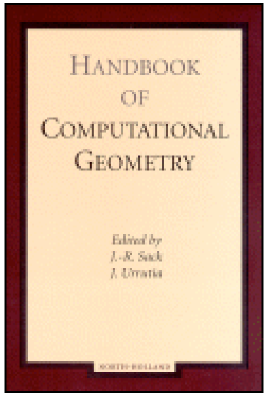


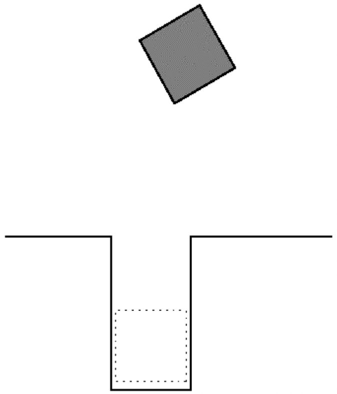




- The 80's

- Special cases and computational geometry



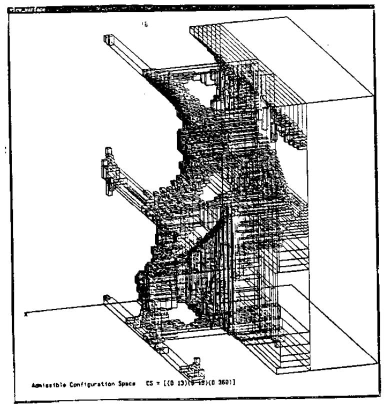


- The 80's: and then?

- No hope for efficient exact and complete solution


- Approximated solution





The 80's: and then?

- No hope for efficient exact and complete solution
- Potential fields and optimization methods

When hardware renewed the topic…

- The 90’s: the amazing efficiency of empirism!

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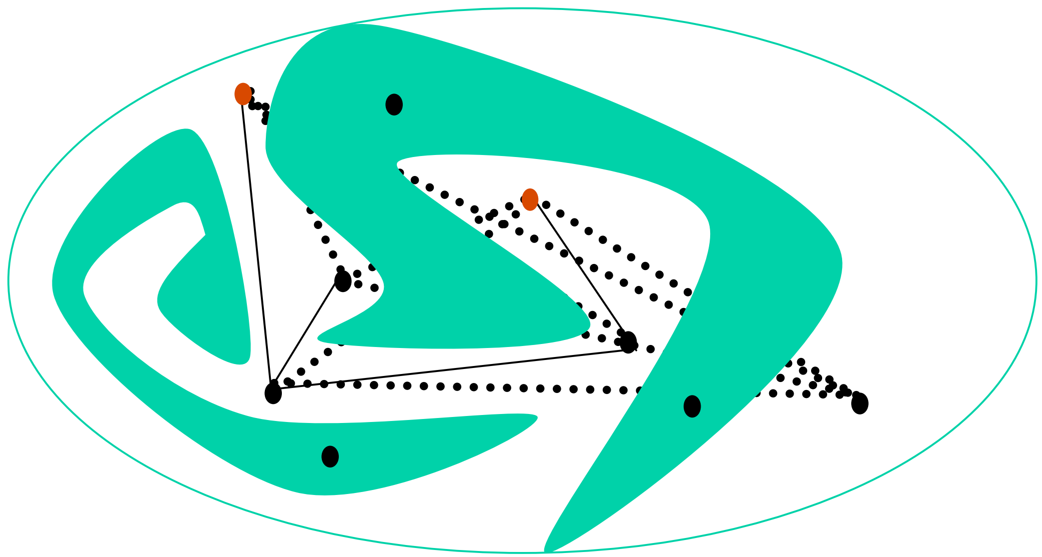
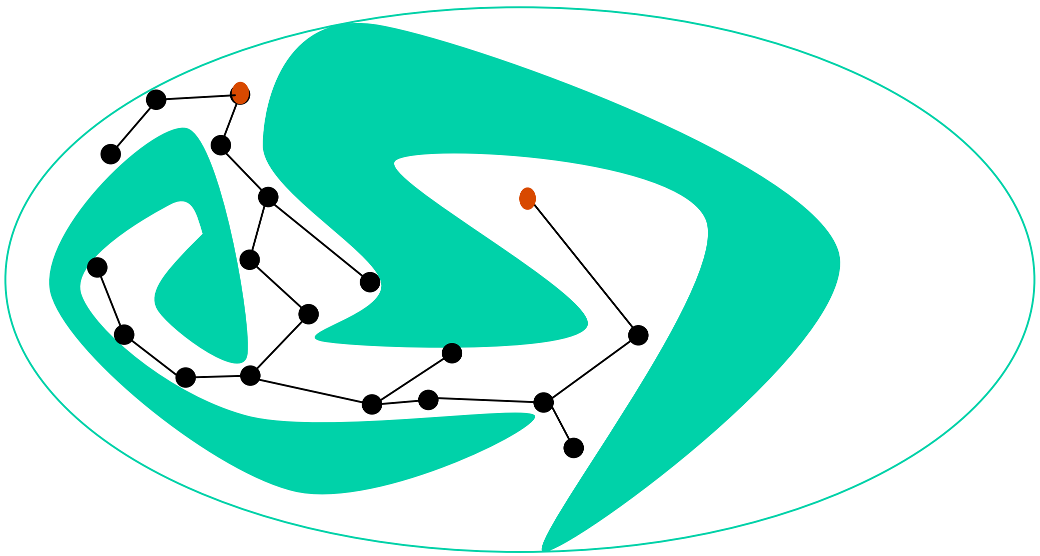


Random sampling

- The 90's: the amazing efficiency of empirism!



Random diffusion

- The 90's: the amazing efficiency of empirism!











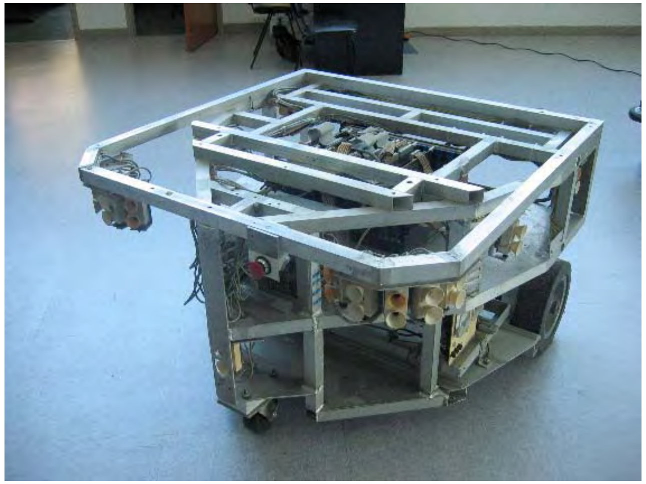




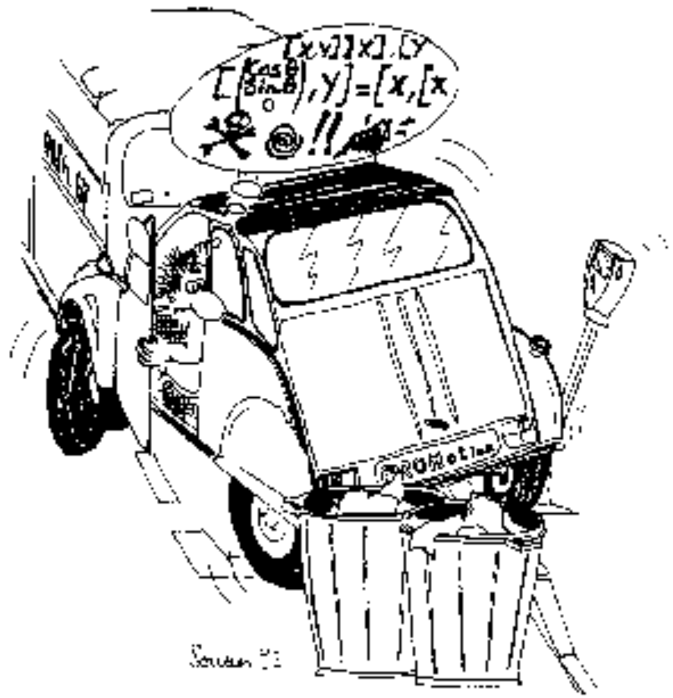


…and that works!

Pause



- The 90's: rolling without sliding!



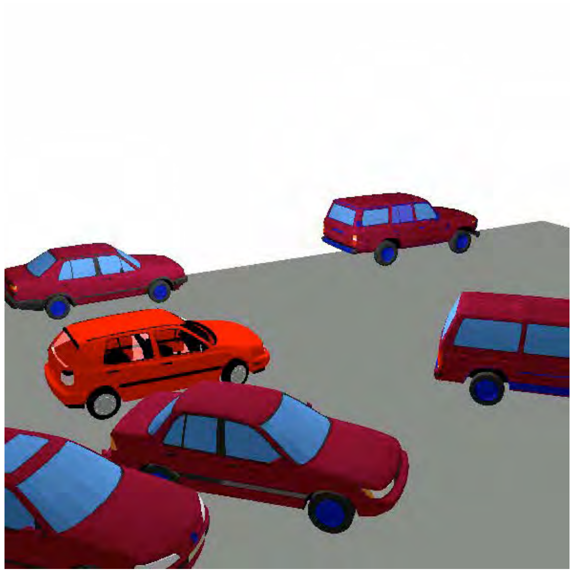
The 90's

- Any admissible motion for the 3D mechanical system appears a collision-free path for a point in the Cspace

- Holonomic systems: converse is **true**

- Nonholonomic ones: converse is **not true**

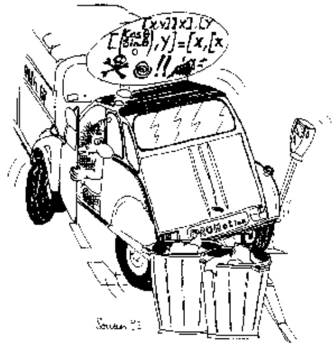
The 90's: the car-like robot



The number of maneuvers varies as the inverse of the square of the size of the free-space

- Holonomy *versus* nonholonomy: an integrability question

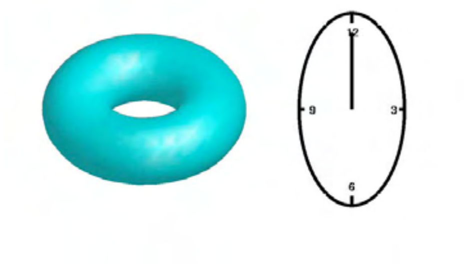
$$\dot{x} = \frac{1}{12}\dot{y}$$

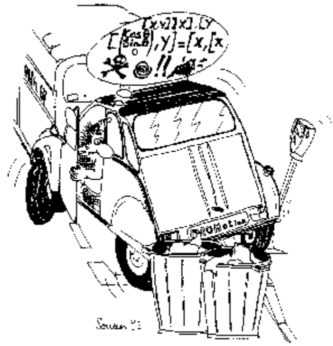


$$\dot{x}\cos\theta - \dot{y}\sin\theta = 0$$

Holonomy *versus* nonholonomy: an integrability question

$$\dot{x} = \frac{1}{12}\dot{y}$$

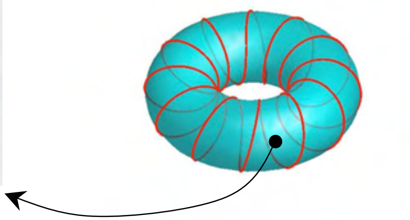




$$\dot{x}\cos\theta - \dot{y}\sin\theta = 0$$

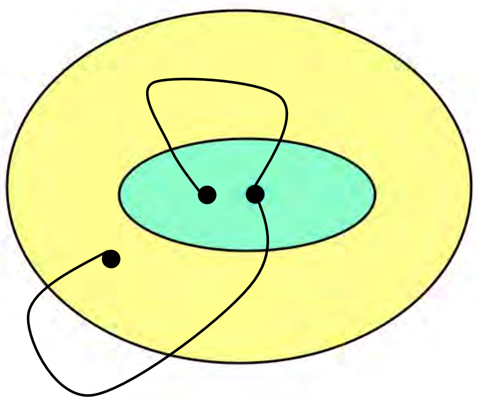
Not integrable

Dim(Reachable(q))=3

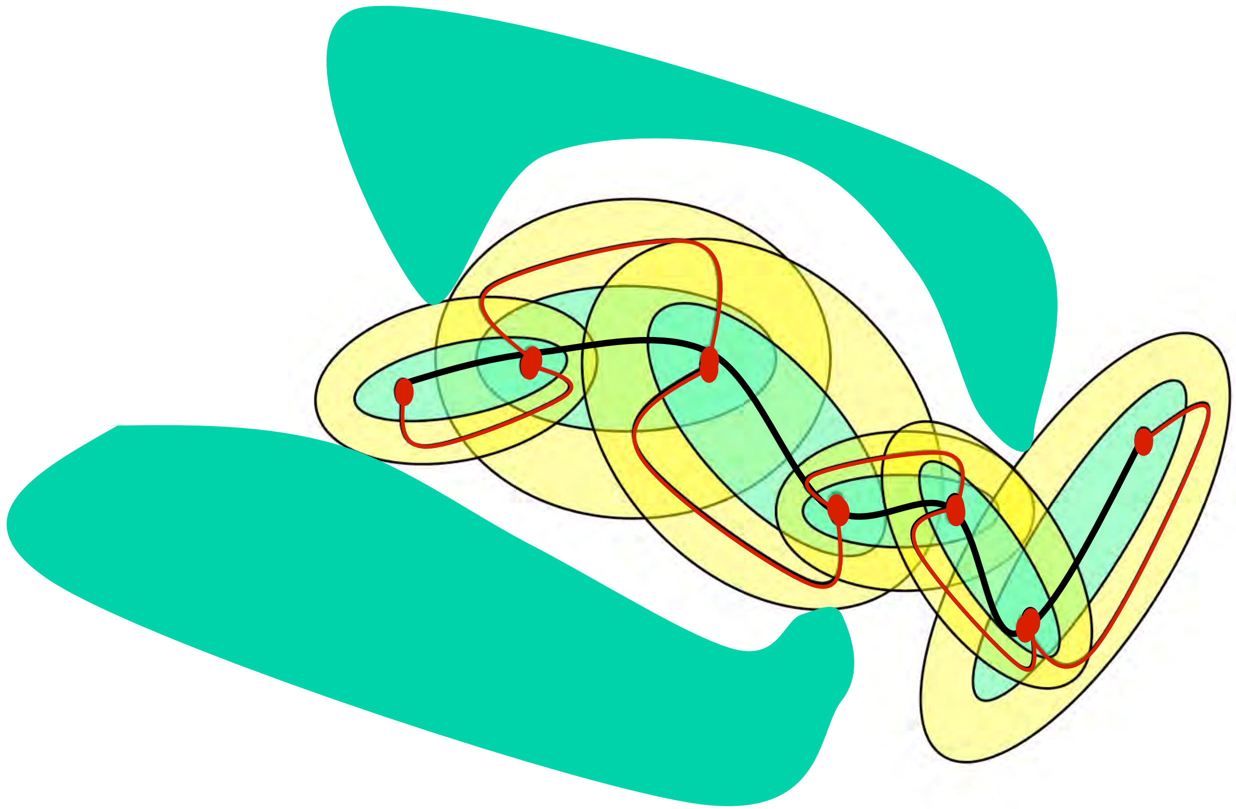


The time that will never happen!!!

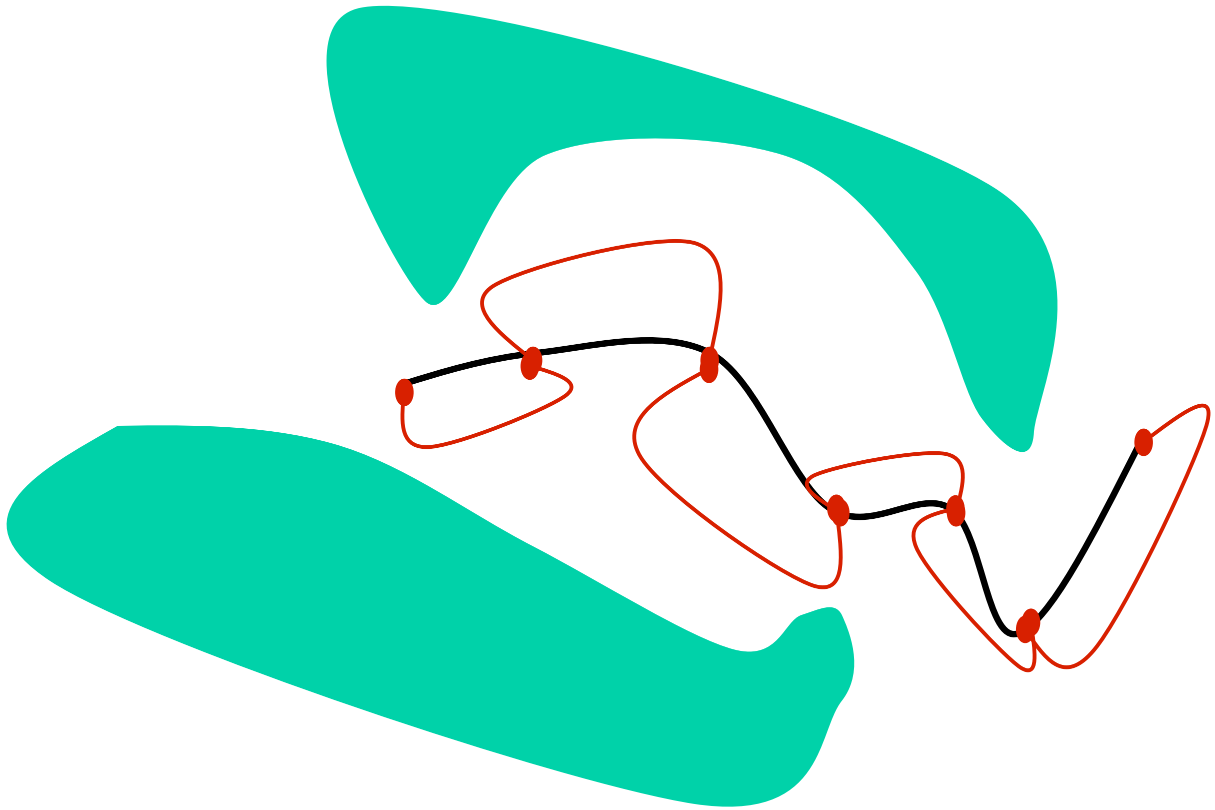
Small-time controllability



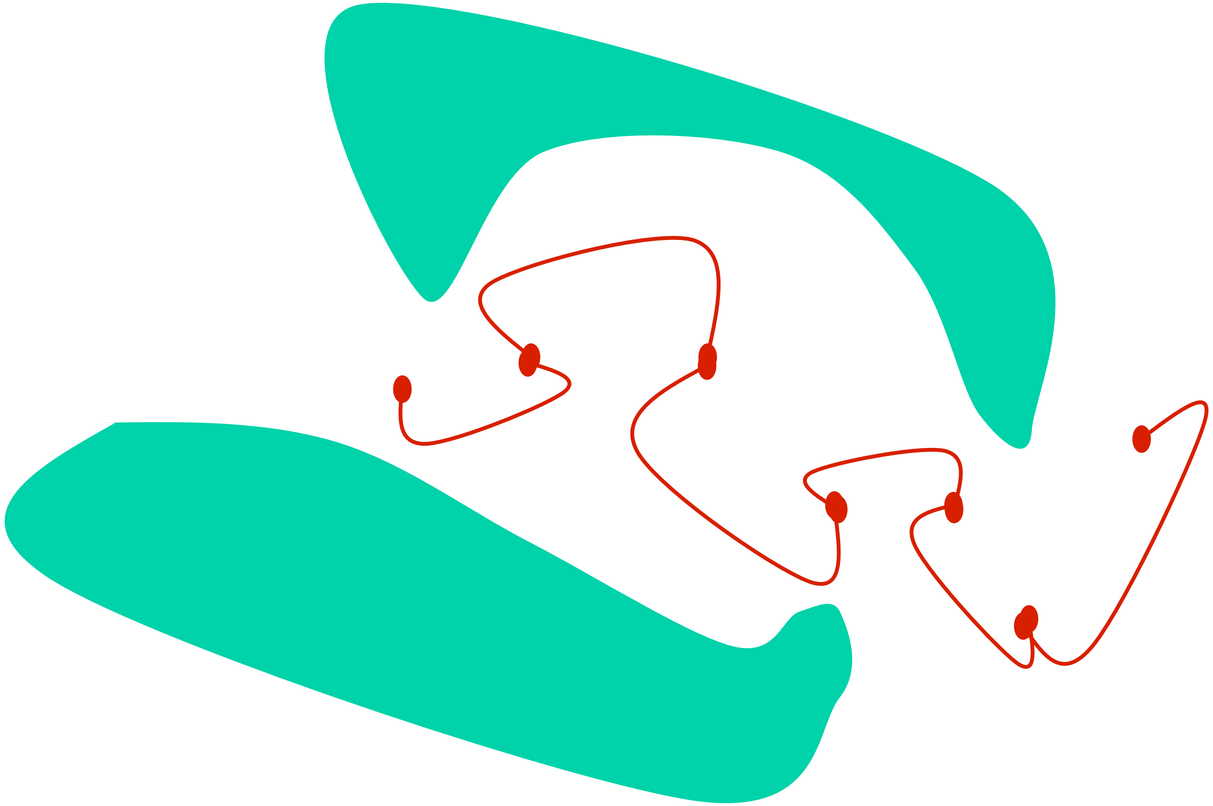
Small-time controllability



Small-time controllability



Small-time controllability



Small-time controllability

- For small-time controllable systems the existence of an admissible motion is characterized by the existence of a path lying in an *open* connected component of free-CSpace

Two central questions

- Is my system nonholonomic ?
- Is my nonholonomic system small-time controllable ?

- Two central questions

- Is my system nonholonomic ?

Frobenius Theorem

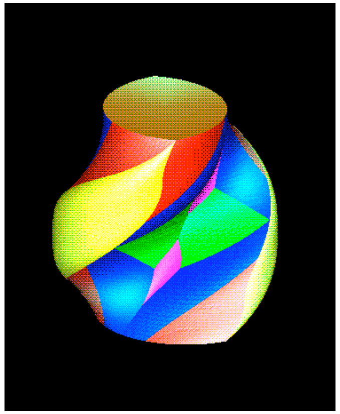
- Is my nonholonomic system small-time controllable ?

Lie Algebra Rank Condition

Nonholonomic systems

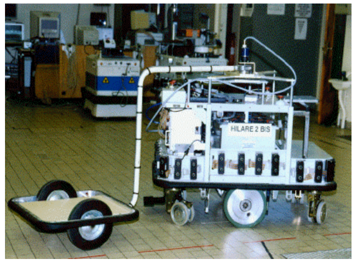
- Devising steering methods that account for small-time controllability

- Optimal control



- Devising steering methods that account for small-time controllability

- Optimal control: numerical approach

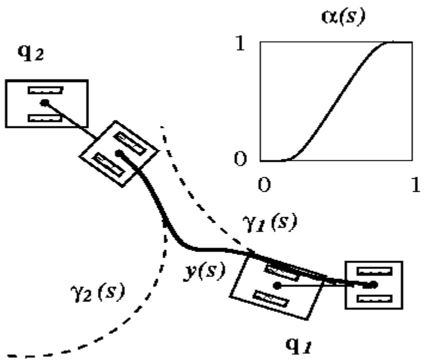




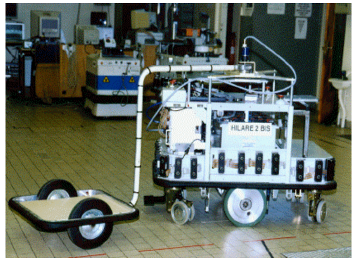
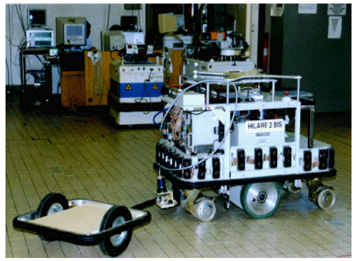


- Devising steering methods that account for small-time controllability

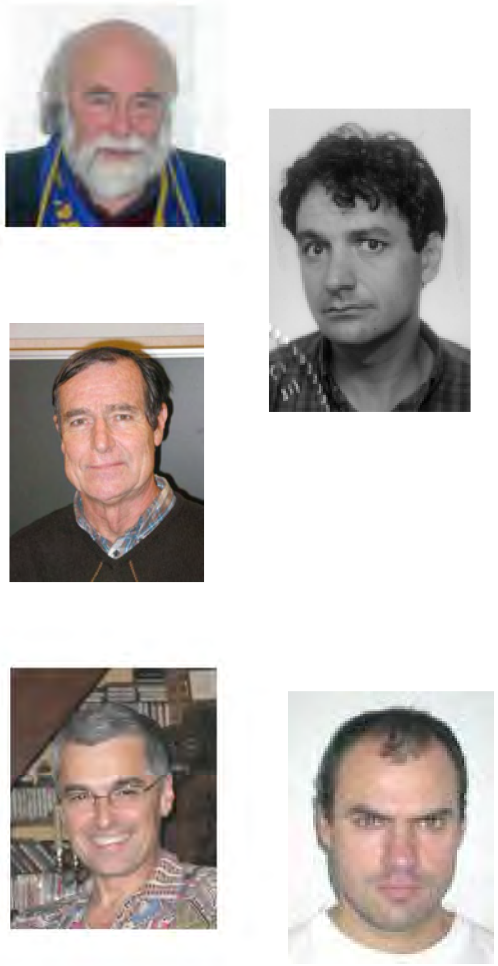
- Chained-from systems
- Flatness

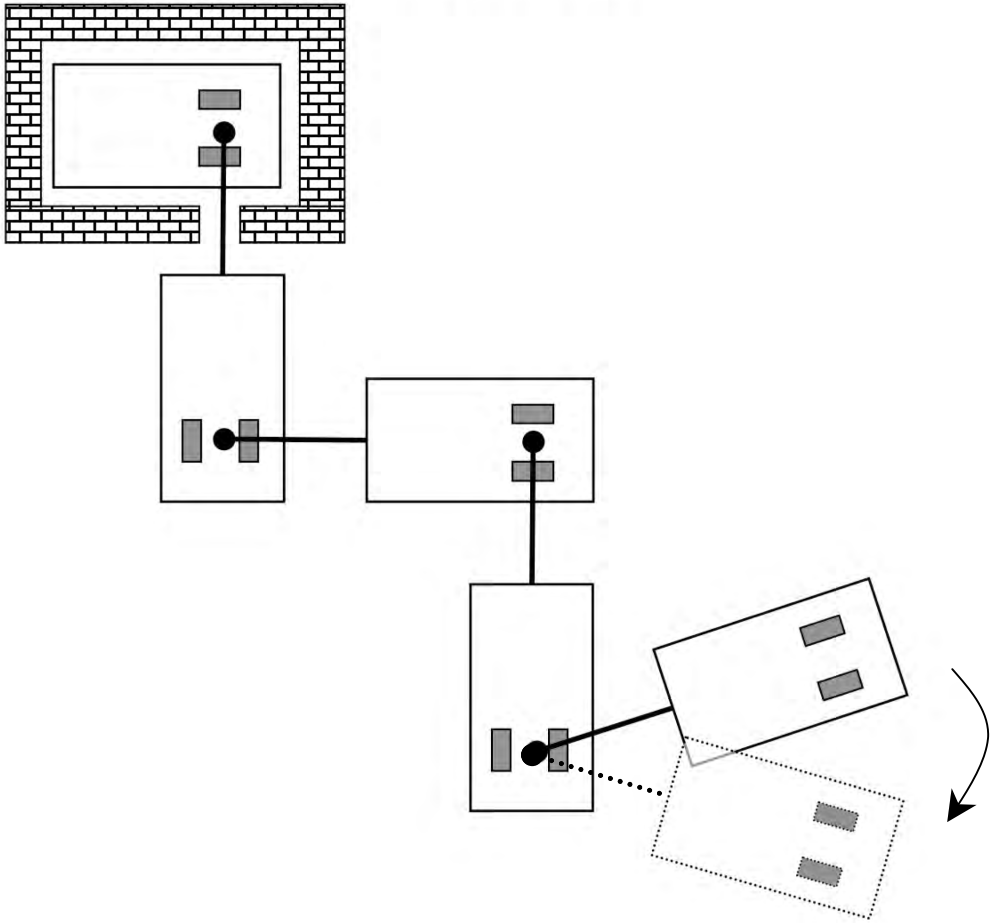


$$y(s) = (1-\alpha(s))\gamma_1(s) + \alpha(s)\gamma_2(s)$$

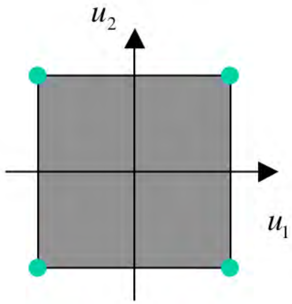
Nonholonomic metrics as a model of computational complexity





Remaining open problems

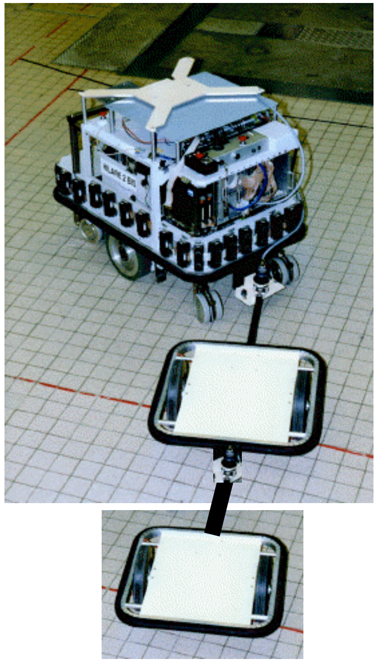
$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\\ \dot{v}_1\\ \dot{v}_2\end{pmatrix} = \begin{pmatrix}1/2(v_1+v_2)\cos\theta\\ 1/2(v_1+v_2)\sin\theta\\ v_1-v_2\\ 0\\ 0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}u_1 + \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}u_2$$



- clothoids
- involutes of a circle

Time-optimal control

Remaining open problems



$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \\ --- \\ --- \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ --- \\ --- \end{pmatrix} u_2 = f_1 u_1 + f_2 u_2$$

Hooking	centered	not
f_1, f_2	2	2
$[f_1, f_2]$	3	3
$[f_1,[f_1,f_2]], [\boldsymbol{f_2},[\boldsymbol{f_1},\boldsymbol{f_2}]]$	4	5
$[f_1,[f_1,[f_1,f_2]]], \ldots$	5	

(2,3,4,5) *versus* (2,3,5)

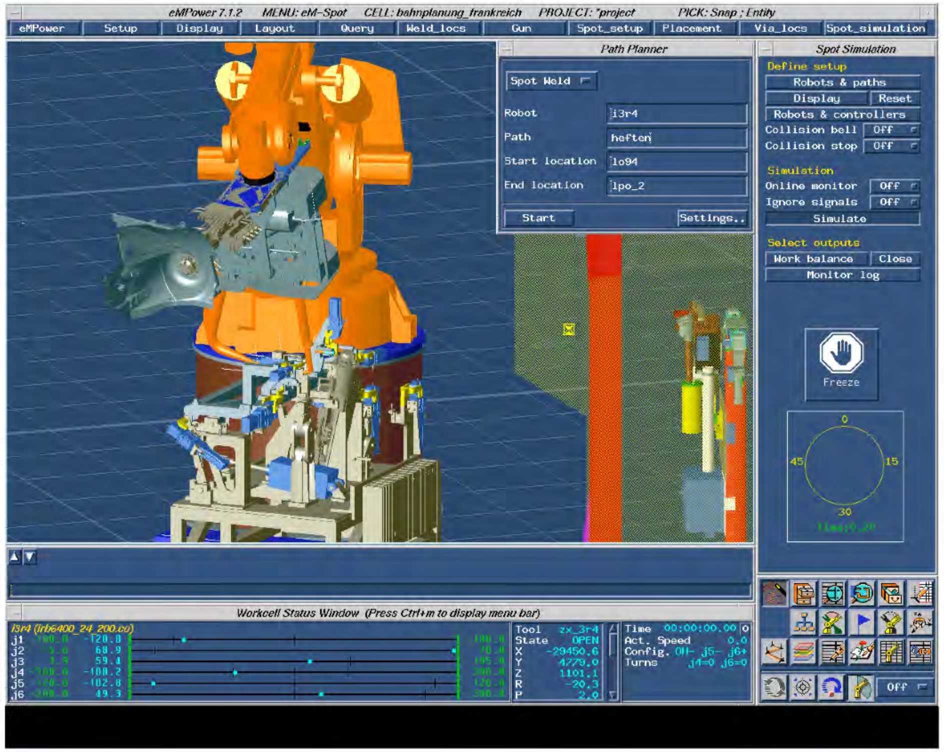
Remaining open problems

Not small-time controllable systems:
is the problem still decidable?

…and that works!

Robot programming

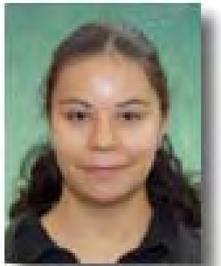
KINEO



Robot motion autonomy

Computer animation

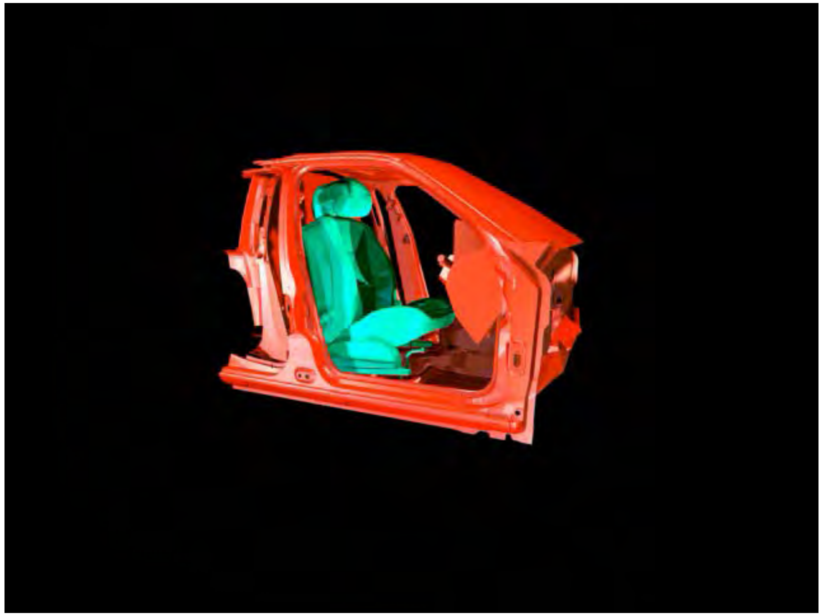
Humanoid robots



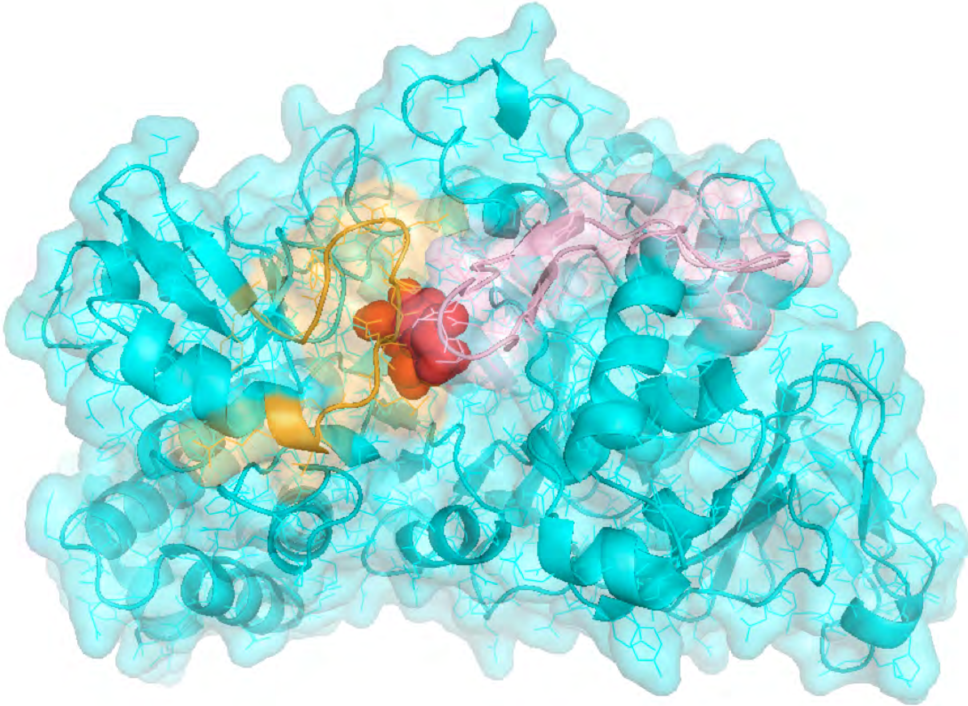
- Computer animation

Product Lifecycle Management

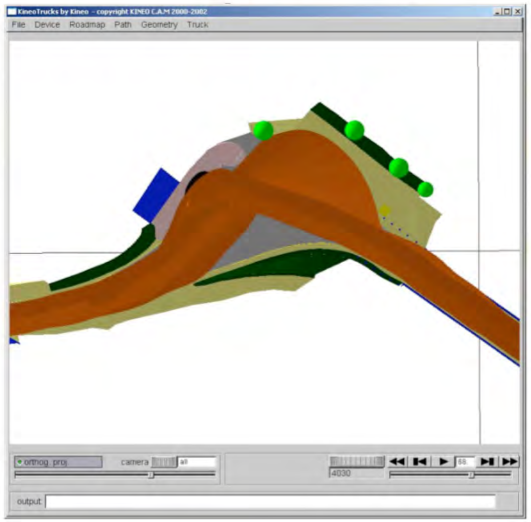
KINEO



Molecular modeling



A380 component transportation by road



They were right !!...

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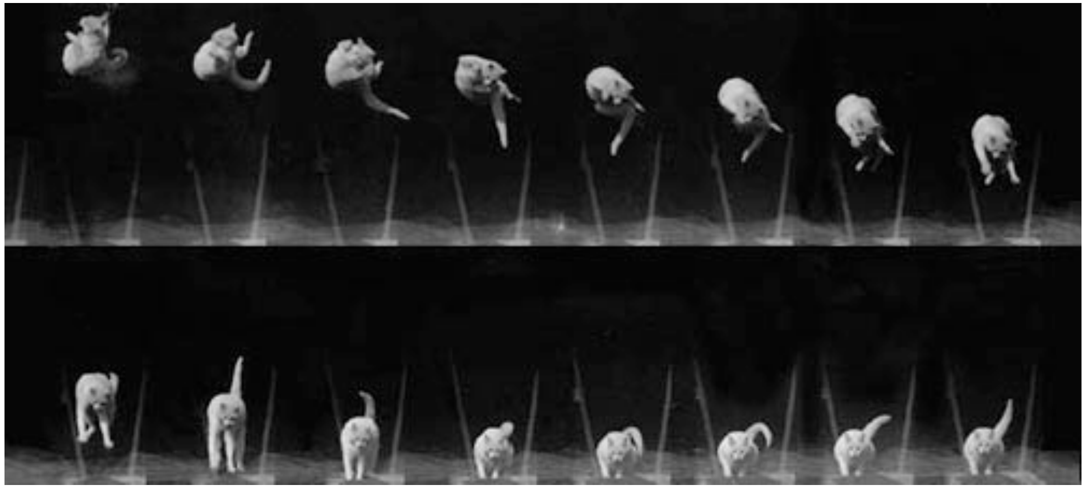
RÉPUBLIQUE FRANÇAISE
Ministère de l'Équipement, des Transports et du Logement

AIRBUS

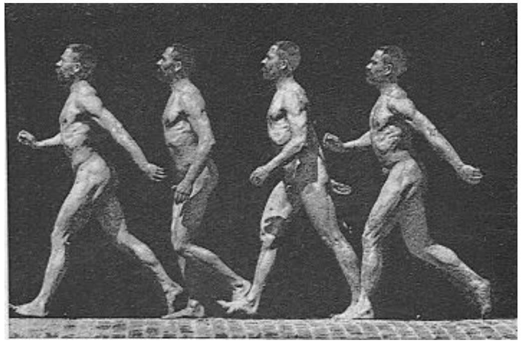
LAAS

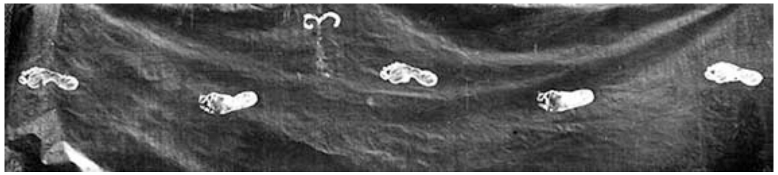
KINEO

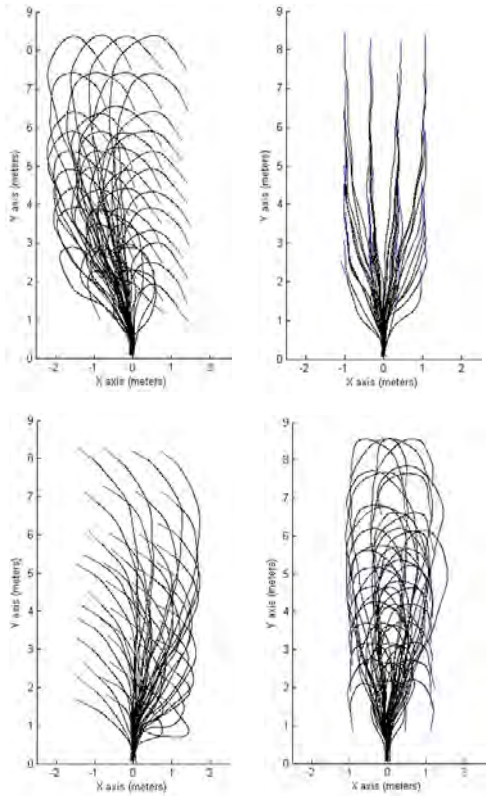
Life Sciences



Life Sciences







As a conclusion…

« Faire ***et*** comprendre »

- Human being modeling