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Jean Lagasse

Quelques souvenirs

L'Automatique
et
La crise énergétique


Un champ infini de recherche et d’application pour l’automatique

Amélioration de l'efficacité et réduction de la pollution par modélisation et contrôle des instabilités de combustion

Improving efficiency and reducing pollution by modelling and control of combustion instabilities

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Scientific Problems: Advances and Challenges

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Outline

- Introduction (Thermo-acoustic instability)
- A realistic combustion instability model for control
- Krylov-Bogoliubov method for analysis of near conservative systems
- Illustrative example (K-B approximation and multiplicative control)
- Combustion model analysis
- Quenching oscillations in combustion instabilities (nonlinear feedback)
- Conclusion

Thermo-acoustic instability

- **Improvement of efficiency and pollution reduction require to use low fuel-air ratio in combustion processes.**

-**However at low fuel-air ratio instabilities occur in combustion processes**

-***Problem :*** **How to get a stable operation at low fuel-ratio?**
- *Option 1*: passive damping (re-design of the combustion chamber)
- *Option 2* : active damping (control designed to quench oscillations)

- To treat the active damping (oscillation quenching) one needs:
    - a relevant tractable analytical model
    - a method of analysis
    - a control variable
    - a control strategy

Thermo-acoustic instability

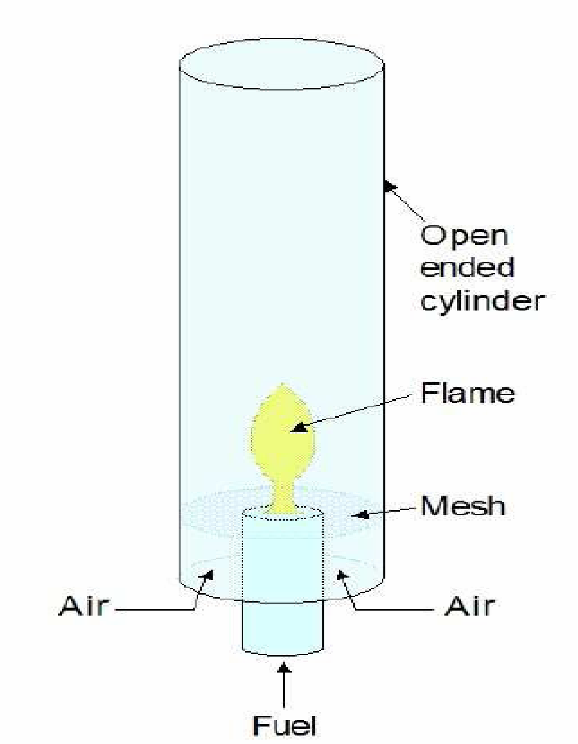
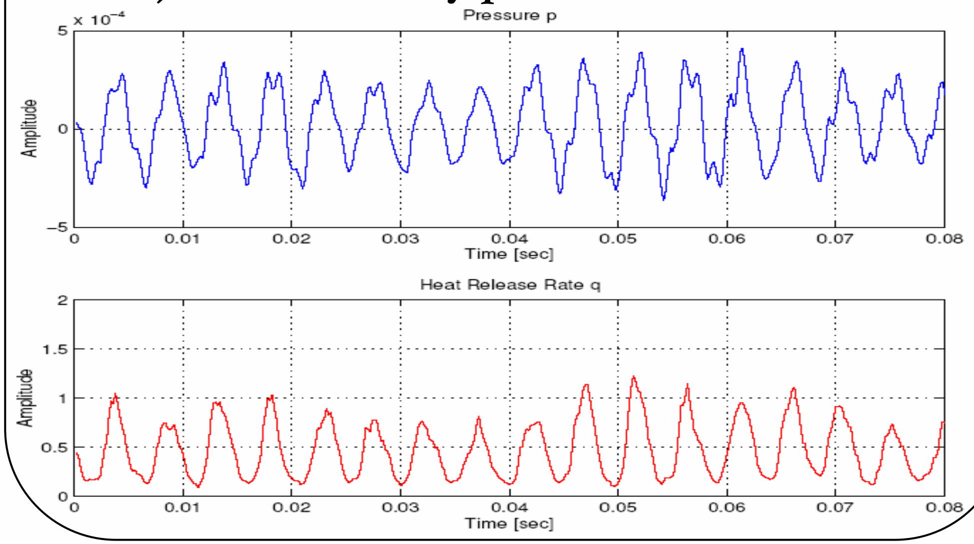


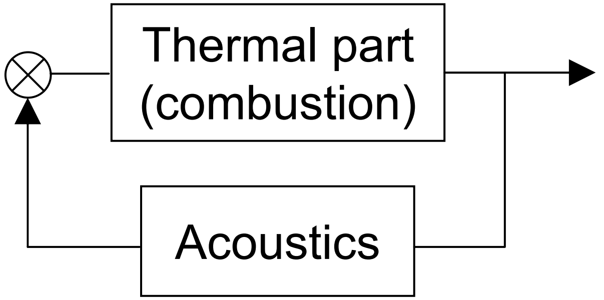
Figure 1: Rijke tube.

Physical interpretation :
an unsteady flame generates pressure wave which are reflected by physical boundaries into the combustion process

At low equivalence ratio (low fuel-to-air ratio) => Instability phenomenon



System interpretation : feedback structure of the phenomenon



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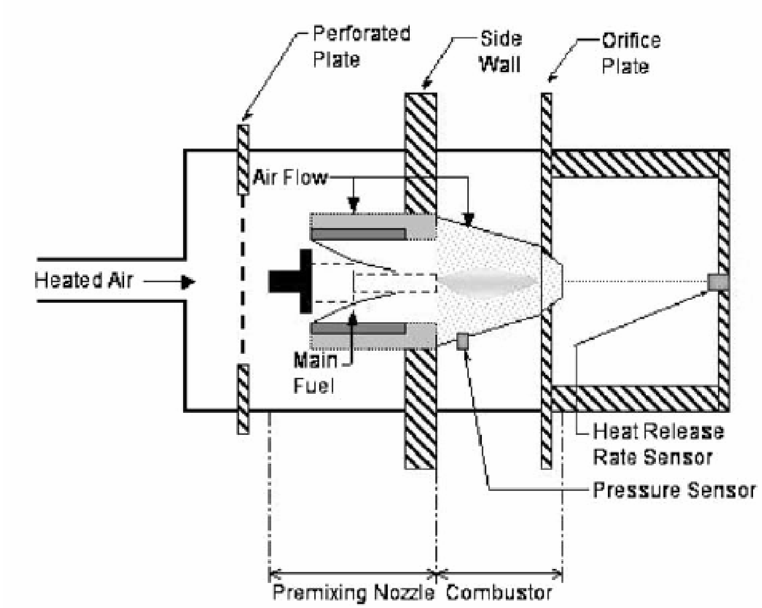
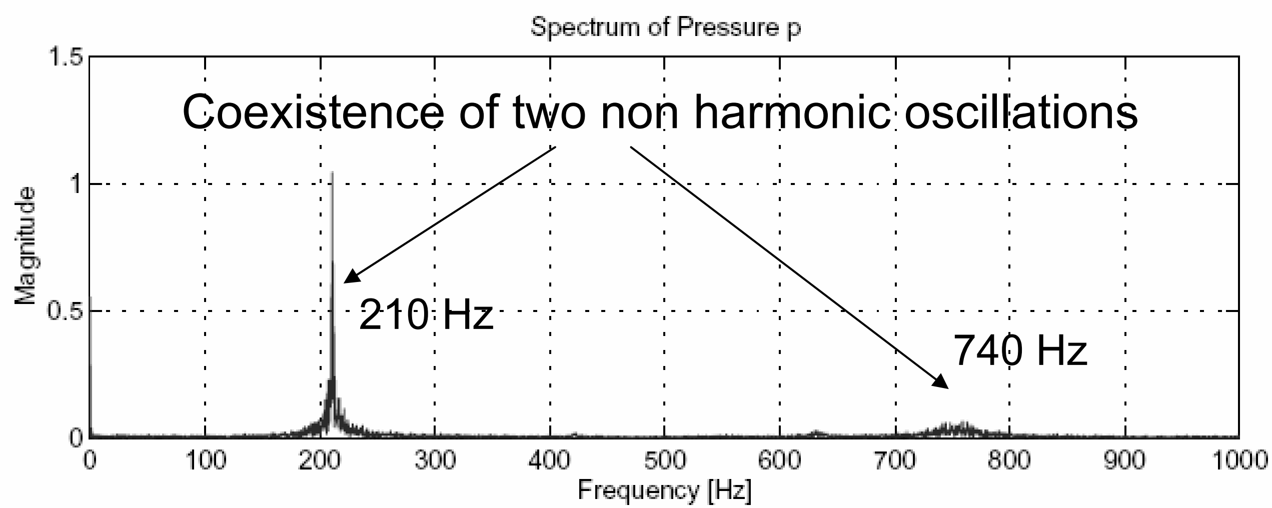
Experimental setting



UTRC/DARPA (United Technology)
Single nozzle rig (gas turbine engine)

The most surprising experimental result !



Practical observations

(Joint work University of California, S. Diego/United Technologies,)

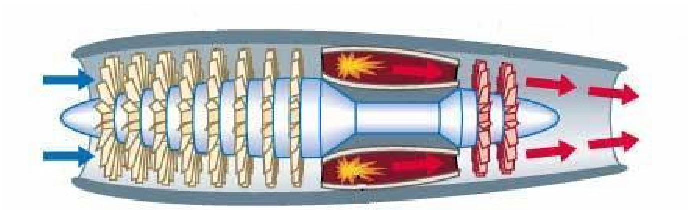
References: Dustan et al. CEP, vol 9 (2001) pp. 1301. Dustan Thesis, UCSD, 2003

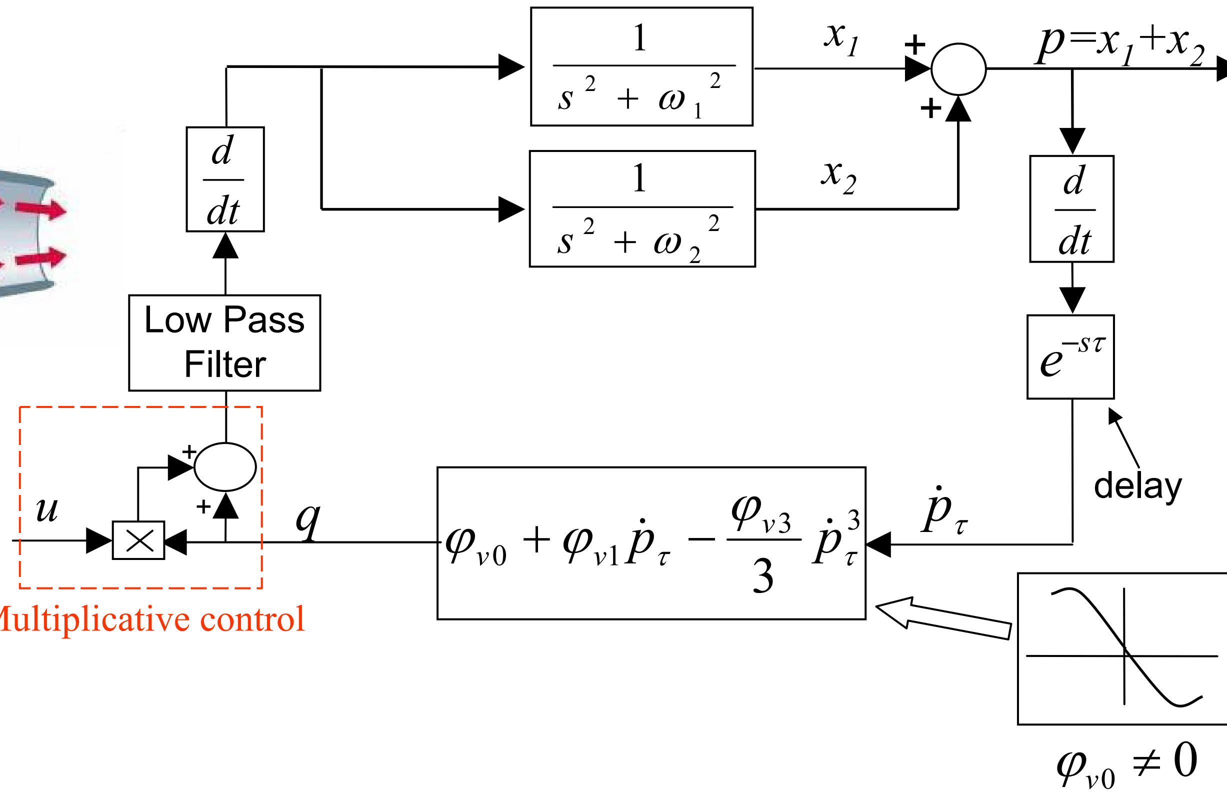
- As the ratio fuel/air decrease, combustion instability appears and manifests itself through oscillations.
- Different operation regimes are possible (single or double oscillations).
- Specific phenomenon : **Coexistence of two non-harmonic frequencies**.
- A strong dominant tone at 210Hz and a weak but persistent non-harmonic tone at 740Hz. $(\omega_2 \approx 3.5\omega_1)$
- Quenching oscillations effect observed through modulation of a fraction of the fuel flow into the combustion chamber (multiplicative control).
- The nonlinear static characteristic present in the dynamic feedback path has been characterized and identified.

Challenges

- Development of a relevant control model and identification
- Analytical demonstration that the model can reproduce the various observed phenomena
- Analytical design of model based feedback quenching strategies using the fuel flow modulation as a multiplicative control input

Combustion instability control model – a "grey" model approach





p – pressure
q – heat release rate
τ – delay
u – fuel modulation
LPF- low pass filter
(comparing to natural frequencies)

Note : The delay exists and is important for explaining the phenomena observed in practice

$$\begin{cases} \ddot{x}_1 + \omega_1^2 x_1 = \frac{d}{dt} LPF\left\{(1+u)\left(\varphi_{v0} + \varphi_{v1}\dot{p}_\tau - \frac{\varphi_{v3}}{3}\dot{p}_\tau^3\right)\right\}, \\ \ddot{x}_2 + \omega_2^2 x_2 = \frac{d}{dt} LPF\left\{(1+u)\left(\varphi_{v0} + \varphi_{v1}\dot{p}_\tau - \frac{\varphi_{v3}}{3}\dot{p}_\tau^3\right)\right\}. \end{cases}$$

Difficulty : **equations analysis !!**

It is a near conservative system

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Krylov-Bogoliubov (K-B) method (1937)

$$\frac{d^2x}{dt^2}+\omega^2 x=\varepsilon f(x,\frac{dx}{dt}) \qquad (1)$$

$\varepsilon = 0$

a and θ are constant

$$x = a\cos(\omega t+\theta) \qquad (2)$$

$$\frac{dx}{dt} = -\omega a\sin(\omega t+\theta) \qquad (3)$$

$\varepsilon \neq 0$

a and θ are (slowly) time varying functions to be determined

Differentiating (2) : $\frac{dx}{dt}=\frac{da}{dt}\cos(\omega t+\theta)-a\frac{d\theta}{dt}\sin(\omega t+\theta)-\omega a\sin(\omega t+\theta)$

But from (3):

$$\frac{da}{dt}\cos(\omega t+\theta)-a\frac{d\theta}{dt}\sin(\omega t+\theta)=0 \qquad (4)$$

$$\frac{d^2x}{dt^2}=-\frac{da}{dt}\omega\sin(\omega t+\theta)-\omega a\frac{d\theta}{dt}\cos(\omega t+\theta)-\omega^2 a\cos(\omega t+\theta) \qquad (5)$$

$$-\frac{da}{dt}\omega\sin(\omega t+\theta)-\omega a\frac{d\theta}{dt}\cos(\omega t+\theta)=\varepsilon f(a\cos(\omega t+\theta),-\omega a\sin(\omega t+\theta)) \qquad (1')$$

Krylov-Bogoliubov (K-B) method

Multiplying (4) by $\omega\cos(\omega t+\theta)$ and (1') by $-\sin(\omega t+\theta)$

[(4)+(1')]/ω: $$\frac{da}{dt}=-\frac{\varepsilon}{\omega}f\left[a\cos(\omega t+\theta),-\omega a\sin(\omega t+\theta)\right]\sin(\omega t+\theta) \quad (6)$$

Multiplying (4) by $\omega\sin(\omega t+\theta)$ and (1') by $\cos(\omega t+\theta)$

[(4)+(1')]/ωa: $$\frac{d\theta}{dt}=-\frac{\varepsilon}{\omega a}f\left[a\cos(\omega t+\theta),-\omega a\sin(\omega t+\theta)\right]\cos(\omega t+\theta) \quad (7)$$

-Right hand side of (6): instantaneous damping
-Right hand side of (6) and (7): periodic with $T=2\pi/\omega$
- a and θ are slowly time varying with respect to T
-*"averaging" over one period can be used*

Consider: $$f(.,.)=H(a,\theta)\sin(\omega t+\theta)+G(a,\theta)\cos(\omega t+\theta)+R(2\omega,3\omega....)$$

Yelds after averaging over one period T:

$$\frac{da}{dt}=-\frac{\varepsilon}{2\omega}H(a,\theta)$$
$$\frac{d\theta}{dt}=-\frac{\varepsilon}{2\omega a}G(a,\theta)$$

Krylov-Bogoliubov (K-B) method

• For the class of systems called *near-conservative autonomous systems* :

$$\frac{d^2x_j}{dt^2}+\omega_j^2x_j=\epsilon f_j\left(x,\frac{dx}{dt}\right),\quad (j=1,2,\ldots,n)$$

where n is number of resonators

• Method based on the approximation of the general solution by a sinusoidal solution with slowly varying amplitude and phase

$$x_j=a_j\cos(\psi_j),\ \text{with}\ \psi_j=\omega_jt+\theta_j$$

• $f_j(.,.)$ takes the form:

$$f_j\left(a_1\cos(\omega_1t+\theta_1),\ldots,a_n\cos(\omega_nt+\theta_n),-a_1\omega_1\sin(\omega_1t+\theta_1),\ldots,-a_n\omega_n\sin(\omega_nt+\theta_n)\right)$$
$$=H_{jj}\sin(\omega_jt+\theta_j)+G_{jj}\cos(\omega_jt+\theta_j)+\sum_{\omega_j\not\approx\omega_\ell}^{r}\left(H_{\ell j}\sin(\omega_\ell t+\theta_\ell)+G_{\ell j}\cos(\omega_\ell t+\theta_\ell)\right),$$

• a_j and θ_j are obtained from

$$\begin{cases}\frac{da_j}{dt}=-\frac{\epsilon}{2\omega_j}H_{jj}(a_1,\ldots,a_n,\theta_1,\ldots,\theta_n),\\ \frac{d\theta_j}{dt}=-\frac{\epsilon}{2\omega_ja_j}G_{jj}(a_1,\ldots,a_n,\theta_1,\ldots,\theta_n).\end{cases}$$

Rem: $\varepsilon=1$ can be treated making $\lim\varepsilon=1$ in the solution for $\varepsilon<1$ *(one assumes that "f" is small)*

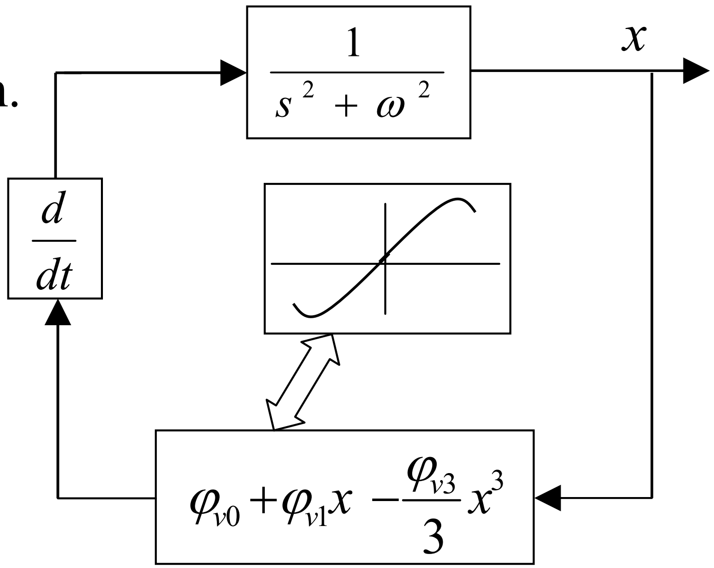
Illustrative example

Example : a generalized van der Pol equation

- *Why?* :

- Proximity between combustion instabilities and occurrence of oscillations in van der Pol equation.
-Good approximation of existing nonlinearity.
- Illustrate the efficiency of K-B method.

Generalized Van der Pol equation



$$\frac{d^2x}{dt^2}+\omega^2 x=\frac{d}{dt}\left\{\varphi_{vo}+\varphi_{v1}x-\frac{\varphi_{v3}}{3}x^3\right\}=\varphi_{v1}\left(1-\frac{\varphi_{v3}}{\varphi_{v1}}x^2\right)\frac{dx}{dt}$$

Original vdP eq.: $\omega=1;\ \varphi_{v1}=\varphi_{v3}=\varepsilon;\ \varphi_{v0}=0$

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Generalized Van der Pol equation

K-B procedure :

- perturbation term :

$$f(x, \frac{dx}{dt}) = \varphi_{v1}\left(1 - \frac{\varphi_{v3}}{\varphi_{v1}}x^2\right)\frac{dx}{dt}$$

- Introducing the sinusoidal solution one gets :

$$f(.,.) = -\varphi_{v1}\left(1 - \frac{\varphi_{v3}}{\varphi_{v1}}a^2\cos^2(\omega t + \theta)\right)a\,\omega\sin(\omega t + \theta)$$

-Using trigonometric relations one gets:

$$f(.,.) = \underbrace{-\omega\varphi_{v1}a\left(1 - \frac{\varphi_{v3}}{4\varphi_{v1}}a^2\right)}_{H}\sin(\omega t + \theta) - \frac{\omega\varphi_{v3}a^3}{4}\sin\left(3(\omega t + \theta)\right)$$

-Consequence :

$$\begin{cases} \dot{a} = \frac{\varphi_{v1}}{2}a\left(1 - \frac{\varphi_{v3}}{4\varphi_{v1}}a^2\right) \\ \dot{\theta} = 0\,. \end{cases} \Longrightarrow$$

Steady oscillations with :

- radian frequency: ω
- amplitude: $2\sqrt{\frac{\varphi_{v1}}{\varphi_{v3}}}$

Illustrative example - quenching

Multiplicative control : Model based on a generalized van der Pol equation

$$\ddot{x}+\omega^2 x=\frac{d}{dt}\left\{\left(1+\Phi(x)\right)\left(\varphi_{v0}+\varphi_{v1}x-\frac{\varphi_{v3}}{3}x^3\right)\right\}$$

- Illustrate the potential effectiveness of the closed-loop multiplicative control.
- Proposed control law (K,ϕ_{v0} same sign):

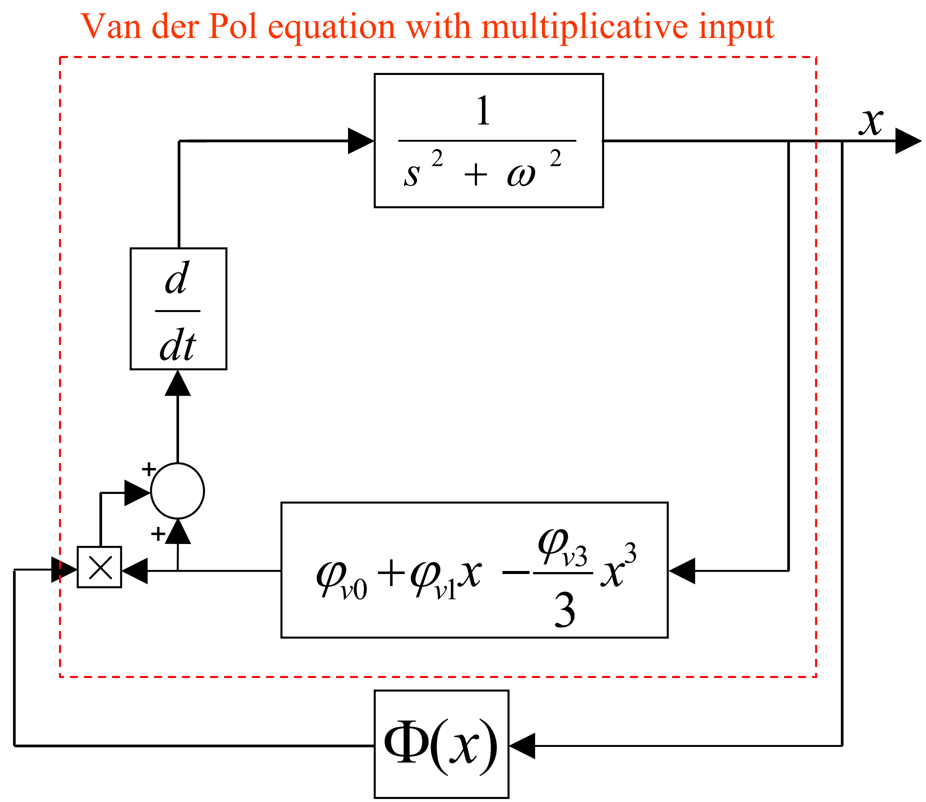
$$\Phi(x)=-Kx-\frac{1}{\varphi_{v0}}\left(\varphi_{v1}x-\frac{\varphi_{v3}}{3}x^3\right)$$

- Establishing the quenching conditions:
  System linearization $(z_1=x;\ z_2=\dot{x})$:

$$A_z=\begin{bmatrix}0 & 1\\ -\omega^2 & -K\varphi_{v0}\end{bmatrix}$$

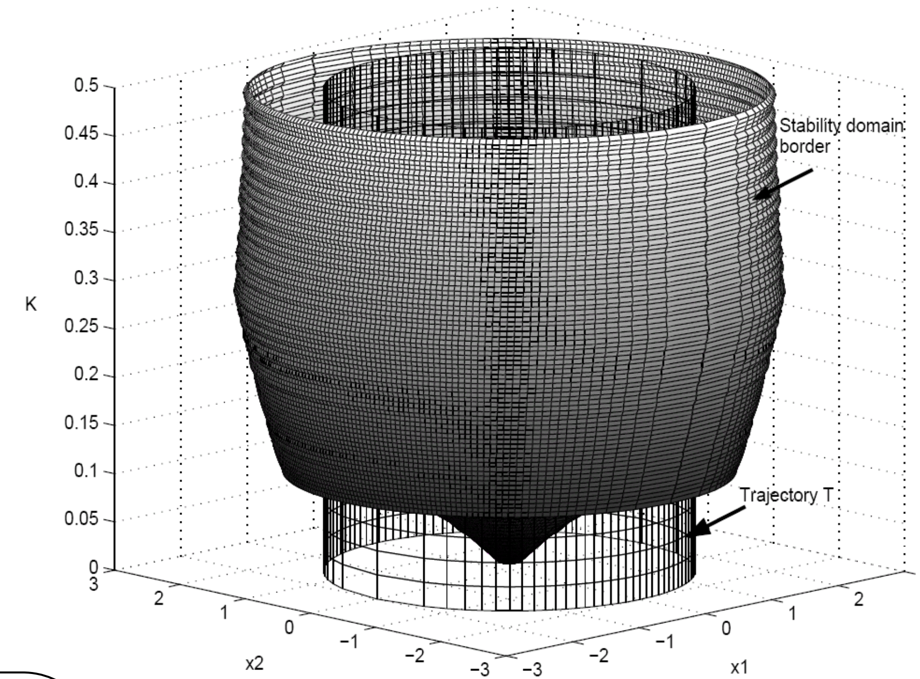
Local stability condition : $K\varphi_{v0}>0$

The quenching of oscillations occurs around the origin in a local domain

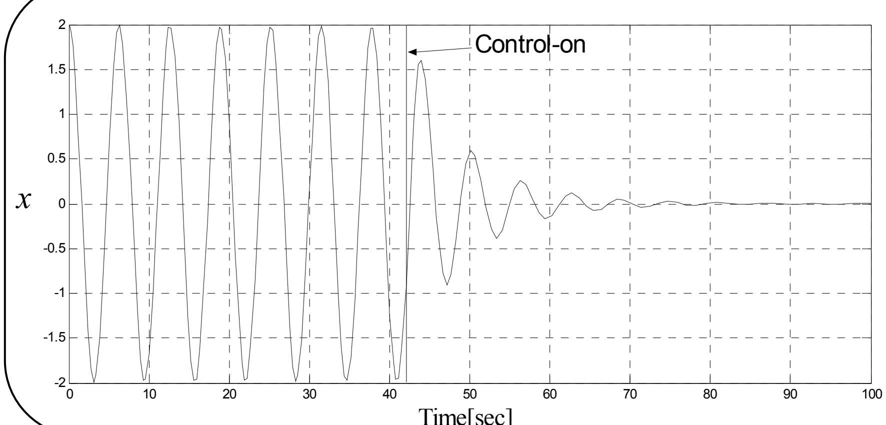


Illustrative example - quenching

Estimation of the local stability domain using a Lyapunov function



Simulation :

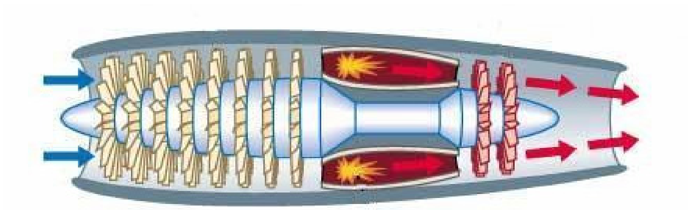


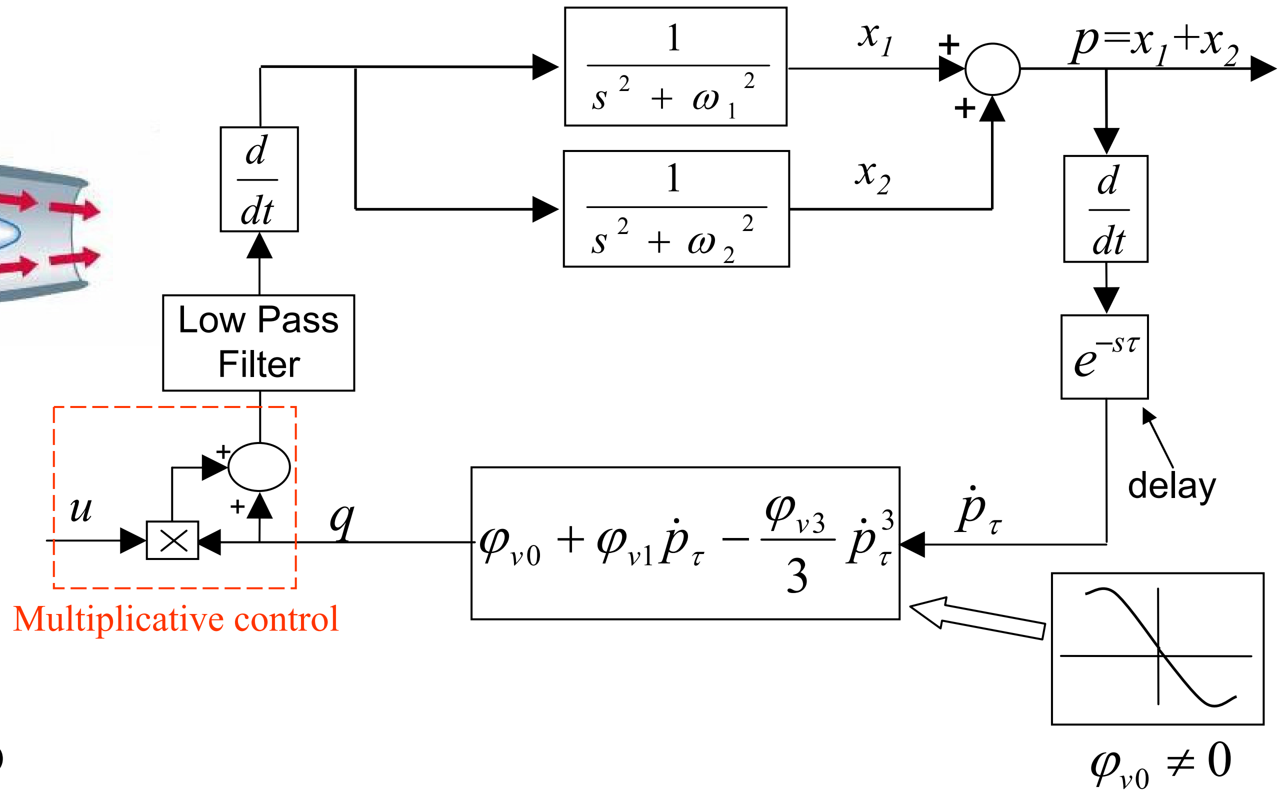
Important !

K-B method gives: $\frac{da}{dt} = -\frac{K\varphi_{v0}}{2}a$

➔ Asymptotical stability meaning that the amplitude of the oscillations goes to zero

Combustion instability model – a ”grey” model approach





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u – fuel modulation
LPF- low pass filter
(comparing to natural frequencies)

Note : The delay exists and is important for explaining the phenomena observed in practice

$$\begin{cases}\ddot{x}_1+\omega_1^2 x_1=\frac{d}{dt}LPF\left\{(1+u)\left(\varphi_{v0}+\varphi_{v1}\dot{p}_\tau-\frac{\varphi_{v3}}{3}\dot{p}_\tau^3\right)\right\},\\ \ddot{x}_2+\omega_2^2 x_2=\frac{d}{dt}LPF\left\{(1+u)\left(\varphi_{v0}+\varphi_{v1}\dot{p}_\tau-\frac{\varphi_{v3}}{3}\dot{p}_\tau^3\right)\right\}.\end{cases}$$

Difficulty : equations analysis !!

Model Analysis

For proposed model ($u=0$) **:**

$$
\begin{aligned}
f_1 = f_2 &= f(x_1, x_2, \frac{dx_1}{dt}, \frac{dx_2}{dt}) \\
&= \frac{d}{dt} LPF \left\{ \varphi_{v0} + \varphi_{v1} \dot{p}_\tau - \frac{\varphi_{v3}}{3} \dot{p}_\tau^3 \right\}, \quad (p = x_1 + x_2).
\end{aligned}
$$

Assumptions :

- A1) Assumption on the delay τ (small compared to the speed of variations of a and q) :

$$
\begin{cases}
a_{i\tau} = a_i - (a_i - a_{i\tau}) \approx a_i, \\
\theta_{i\tau} = \theta_i - (\theta_i - \theta_{i\tau}) \approx \theta_i.
\end{cases}
\quad (i = 1, 2)
$$

- A2) Assumption on low pass filtering (*LPF)* :
  The filter is linear and its dynamic is much faster than the evolution of amplitudes and phases.

- A3) Assumption on validity of the K-B approximation: If amplitudes are asymptotically locally (globally) stable at the origin, then the original system is asymptotically locally stable at the origin.

Model Analysis (2)

- $f_i\left(a_1\cos(\omega_1 t+\theta_1), a_2\cos(\omega_2 t+\theta_2), -a_1\omega_1\sin(\omega_1 t+\theta_1), -a_2\omega_2\sin(\omega_2 t+\theta_2)\right)$

  $\Rightarrow W = \{\omega_1, \omega_2, 3\omega_1, 3\omega_2, 2\omega_1+\omega_2, \omega_1+2\omega_2, 2\omega_1-\omega_2, 2\omega_2-\omega_1\}$

- **Three cases relating the proximity of the natural frequencies :**

1. $\omega_1 \approx \omega_2$: two generators with competitive quenching
2. $\omega_1 \approx 3\omega_2$ $(\omega_2 \approx 3\omega_1)$: mutual synchronization with close frequencies :
3. $\omega_1 \not\approx \{\omega_2, 3\omega_2, \frac{\omega_2}{3}\}$: **the most interesting practical situation**
   (non harmonic frequencies)

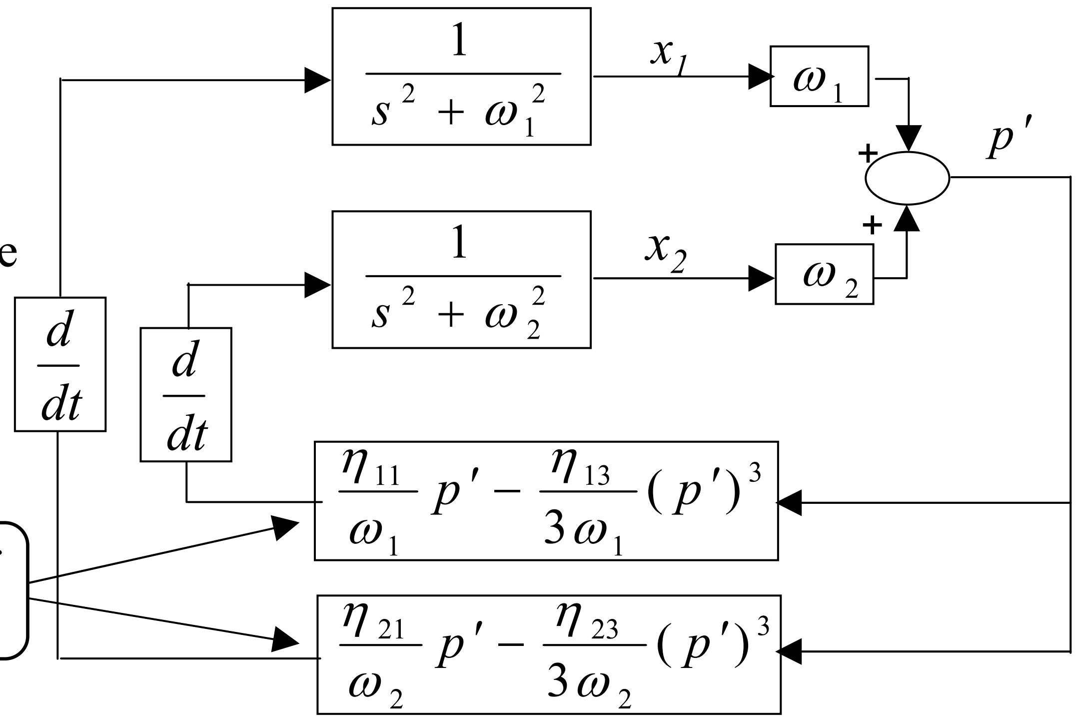
- **K-B Approximation of model for case (3)** :

$$\begin{cases} \dot{a}_1 = \frac{\eta_{11}}{2}a_1 - \frac{\eta_{13}}{2}a_1\left(\frac{(\omega_1 a_1)^2}{4} + \frac{(\omega_2 a_2)^2}{2}\right), \\ \dot{a}_2 = \frac{\eta_{21}}{2}a_2 - \frac{\eta_{23}}{2}a_2\left(\frac{(\omega_2 a_2)^2}{4} + \frac{(\omega_1 a_1)^2}{2}\right). \end{cases}$$

where $\eta_{11}, \eta_{21}, \eta_{13}, \eta_{23}$ are function of : phases introduced by delay and filter, non linearity coefficients and natural resonance frequencies.

- **Equivalent block diagram : Proximity with van der Pol equation**

$\eta_{11}, \eta_{13}, \eta_{21}, \eta_{23}$: coefficients of the new non-linear static functions



- **4 steady states solution :**

$$a_1 = 0 \text{ and } a_2 = 0, \qquad \text{(S.1)}$$

$$a_1 = \frac{2}{\omega_1}\sqrt{\frac{\varphi_{v1}}{\varphi_{v3}}} \text{ and } a_2 = 0, \qquad \text{(S.2)}$$

$$a_1 = 0 \text{ and } a_2 = \frac{2}{\omega_2}\sqrt{\frac{\varphi_{v1}}{\varphi_{v3}}}, \qquad \text{(S.3)}$$

$$a_1 = \frac{2}{\omega_1}\sqrt{\frac{\varphi_{v1}}{3\varphi_{v3}}} \text{ and } a_2 = \frac{2}{\omega_2}\sqrt{\frac{\varphi_{v1}}{3\varphi_{v3}}}. \qquad \text{(S.4)}$$

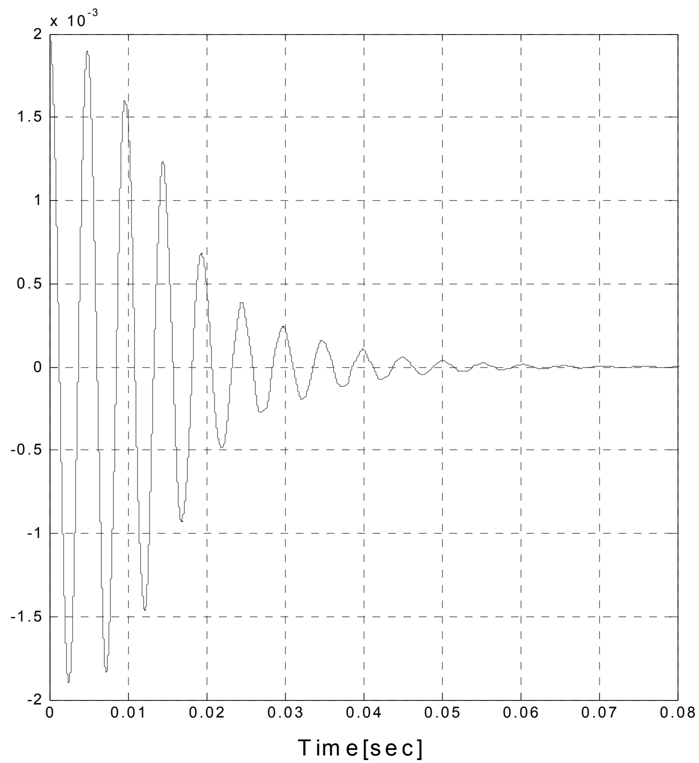
Model Analysis (4)

- (S.1) ⇨ Asymptotically stable system
- Operation conditions :

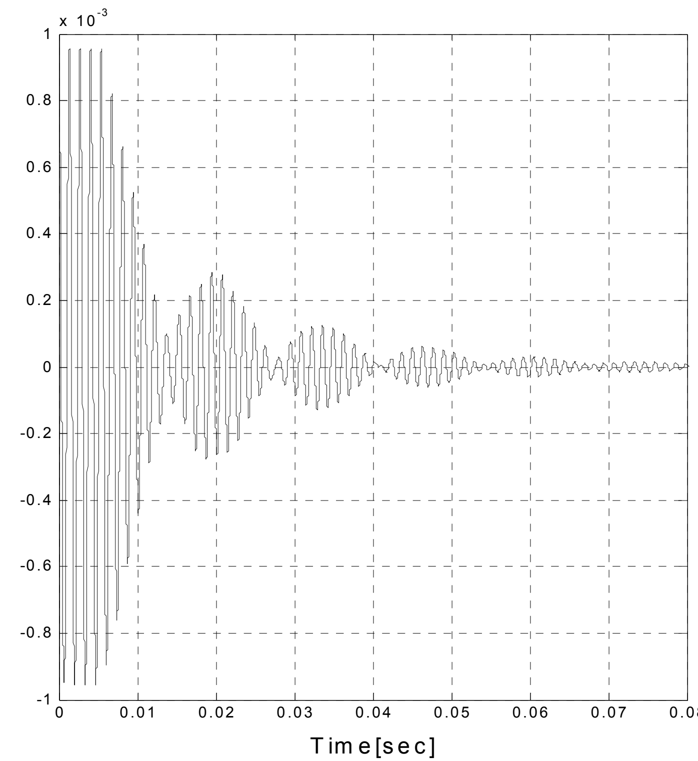
$$\begin{cases} \eta_{11} < 0 \\ \eta_{21} < 0 \end{cases} \qquad \text{(C.1)}$$

- Simulation test:

x1



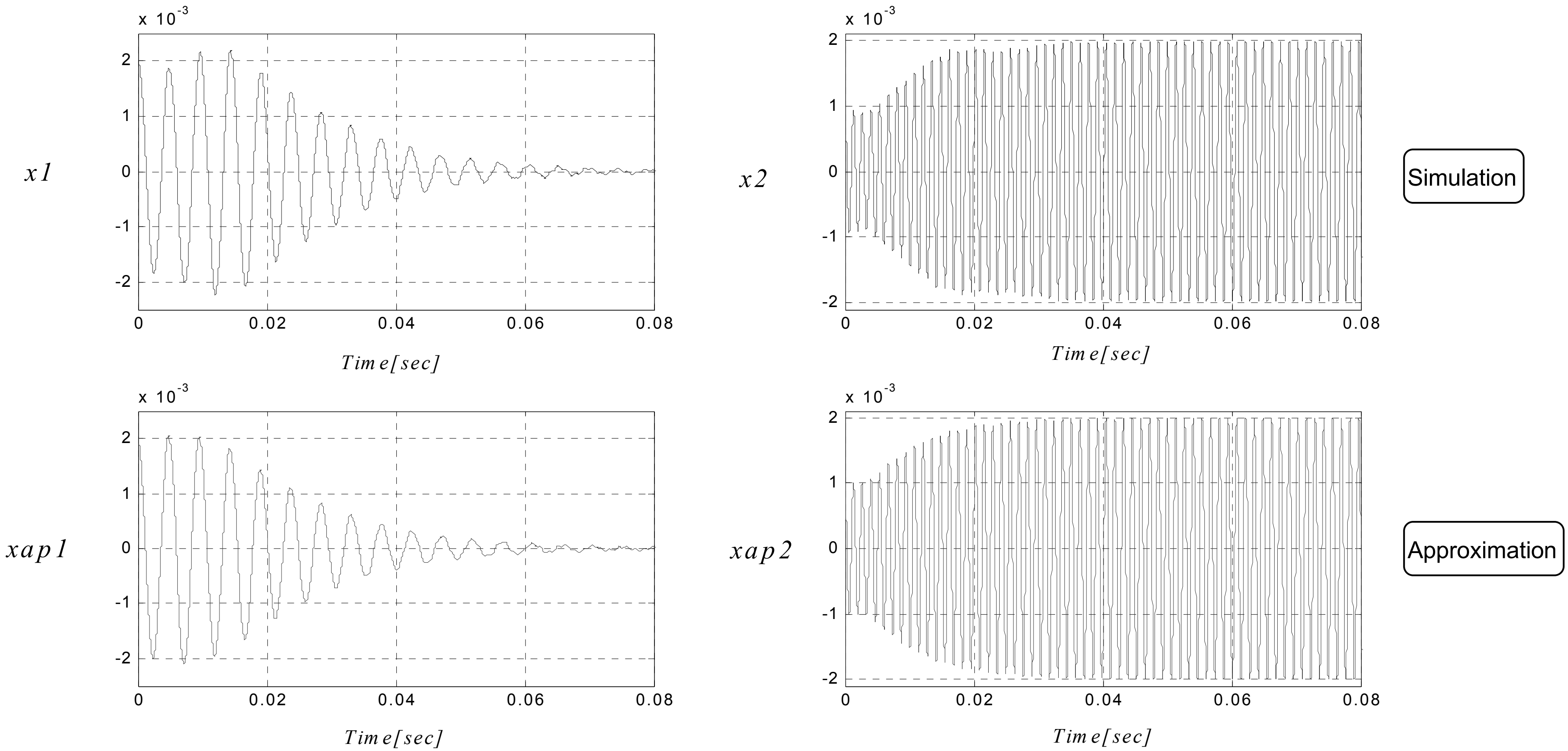
x2



Model Analysis (5)

(S.2) &(S.3) ⇨ Two generators with competitive quenching

- Operation conditions : $\begin{cases} \eta_{11} > 0 \\ 2\eta_{23}\frac{\eta_{11}}{\eta_{13}} - \eta_{21} > 0 \end{cases}$ & $\begin{cases} 2\eta_{13}\frac{\eta_{21}}{\eta_{23}} - \eta_{11} > 0 \\ \eta_{21} > 0 \end{cases}$ (C.2)
- Simulation test:



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Model Analysis (6)

• (S. 4) ⟹ **Simultaneous self-sustained non harmonic oscillations**

Operation conditions **:**

$$\begin{cases} \eta_{13}\left[\frac{2\eta_{21}}{\eta_{23}} - \frac{\eta_{11}}{\eta_{13}}\right] + \eta_{23}\left[\frac{2\eta_{11}}{\eta_{13}} - \frac{\eta_{21}}{\eta_{23}}\right] > 0 \\ \eta_{13}\eta_{23} < 0 \end{cases} \quad \textbf{(C.3)}$$

Particularities :

• Without synchronization

• Conditions on phases introduced by delay and filter : one must be between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, while the other must be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

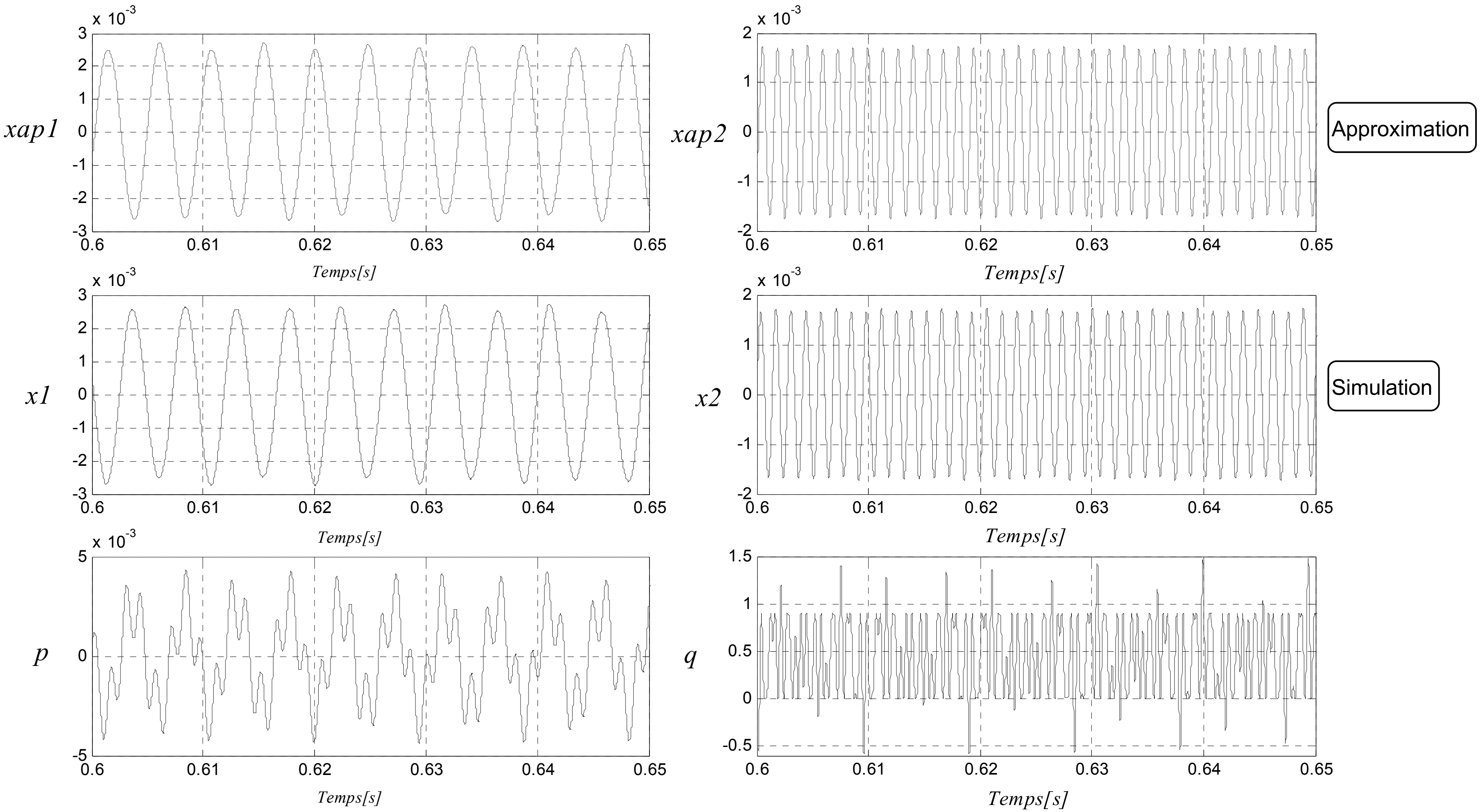
• **When (C.1), (C.2) and (C.3) are not satisfied**

⟹ There does not exist oscillations with stable amplitudes

⟹ Total instability : the oscillations of the system diverge

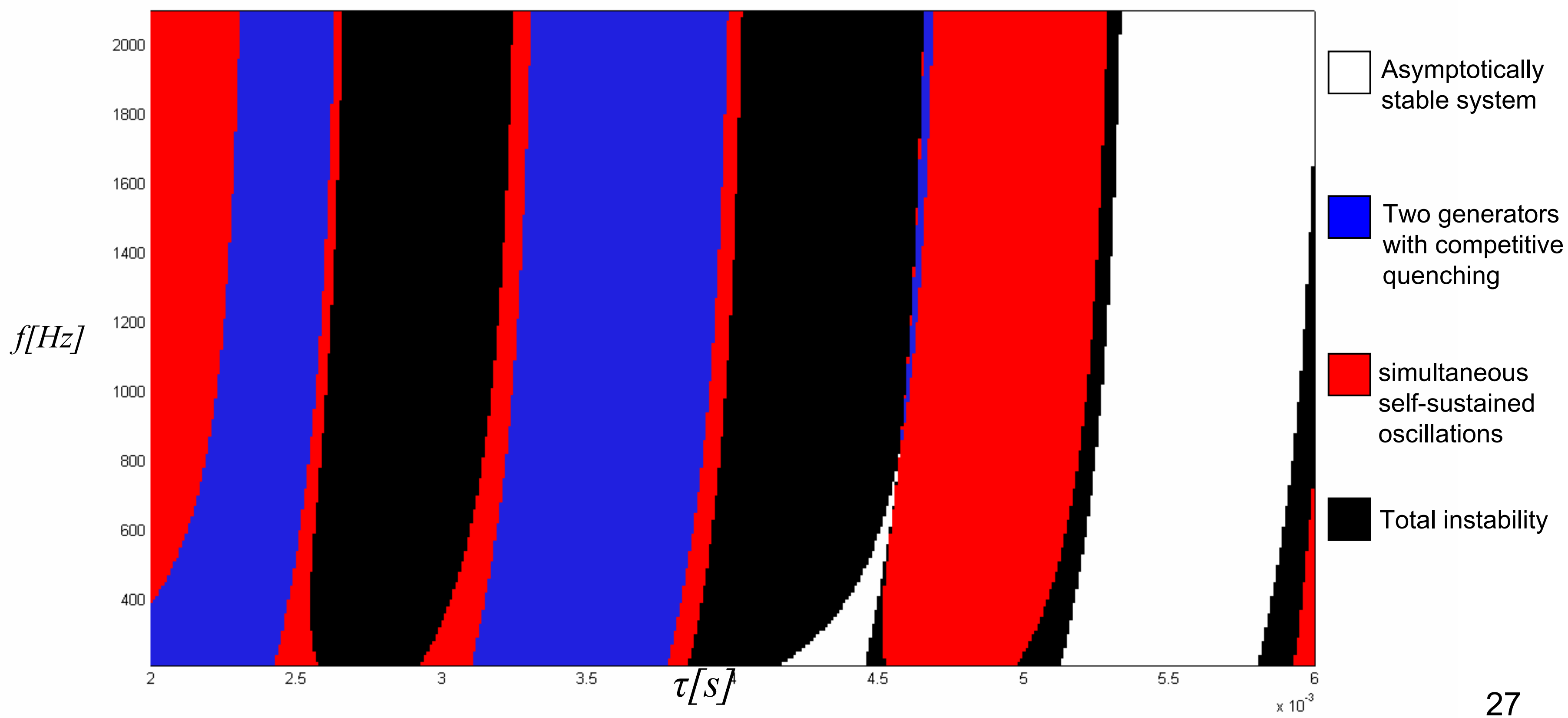
Model Analysis (7)

- **Simulation test of simultaneous self-sustained non harmonic oscillations regime (S.4)**



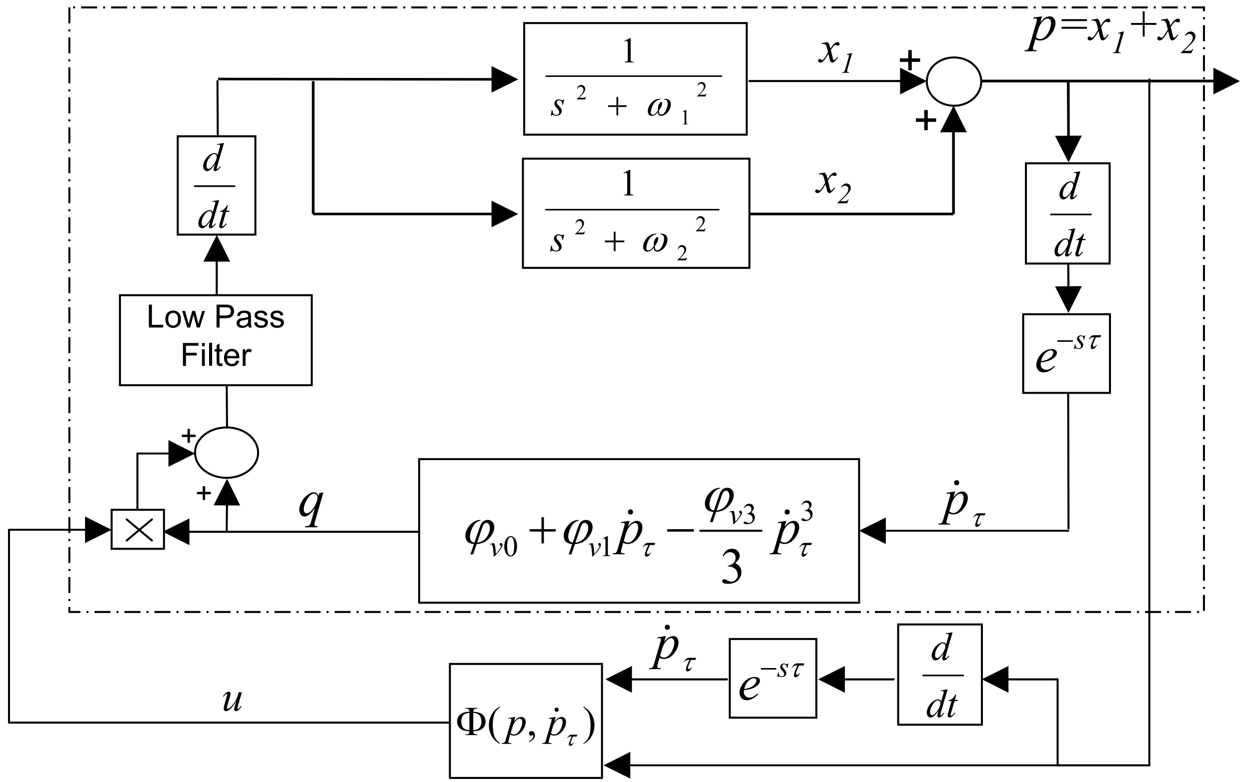
Model Analysis (8)

- The phase domains of conditions (C.1), (C.2) and (C.3) are independent (for a given delay and LPF only one regime is possible)
- **The occurrence of various regimes as a function of the delay and low pass filter (1st order filter) :**



Quenching oscillations in combustion instability (1)

Nonlinear feedback :



Control law:

$$\Phi\left(p,\dot{p}_\tau\right) = -Kp - \frac{1}{\varphi_{v0}}\left(\varphi_{v1}\dot{p}_\tau - \frac{\varphi_{v3}}{3}\dot{p}_\tau^3\right),$$

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Quenching oscillations in combustion instability (2)

Objectives :

- Compensate the physical feedback caused by the coupling between the thermal heat-release process and the acoustics of the combustion chamber
- Direct use of the pressure measurement
- Add positive damping
- Interpretation: Feedback linearization which in addition stabilizes the system

$$\Phi\left(p, \dot{p}_{\tau}\right) = -Kp - \frac{1}{\varphi_{v0}}\left(\varphi_{v1}\dot{p}_{\tau} - \frac{\varphi_{v3}}{3}\dot{p}_{\tau}^{3}\right),$$

(K, φ_{v0} same sign)

Quenching oscillations in combustion instability (3)

- Control law:

$$\Phi\left(p,\dot{p}_\tau\right) = -Kp - \frac{1}{\varphi_{v0}}\left(\varphi_{v1}\dot{p}_\tau - \frac{\varphi_{v3}}{3}\dot{p}_\tau^3\right),$$

- K-B approximations :

$$\begin{cases} \frac{da_1}{dt} = -\frac{1}{2}G(\omega_1)K\varphi_{v0}\cos(\phi(\omega_1))a_1, \\ \frac{da_2}{dt} = -\frac{1}{2}G(\omega_2)K\varphi_{v0}\cos(\phi(\omega_2))a_2. \end{cases}$$

$G(\omega); \phi(\omega)$ = gain and phase of the LPF at frequency ω

K-B linearization

- Global stability conditions of amplitudes at the origin :

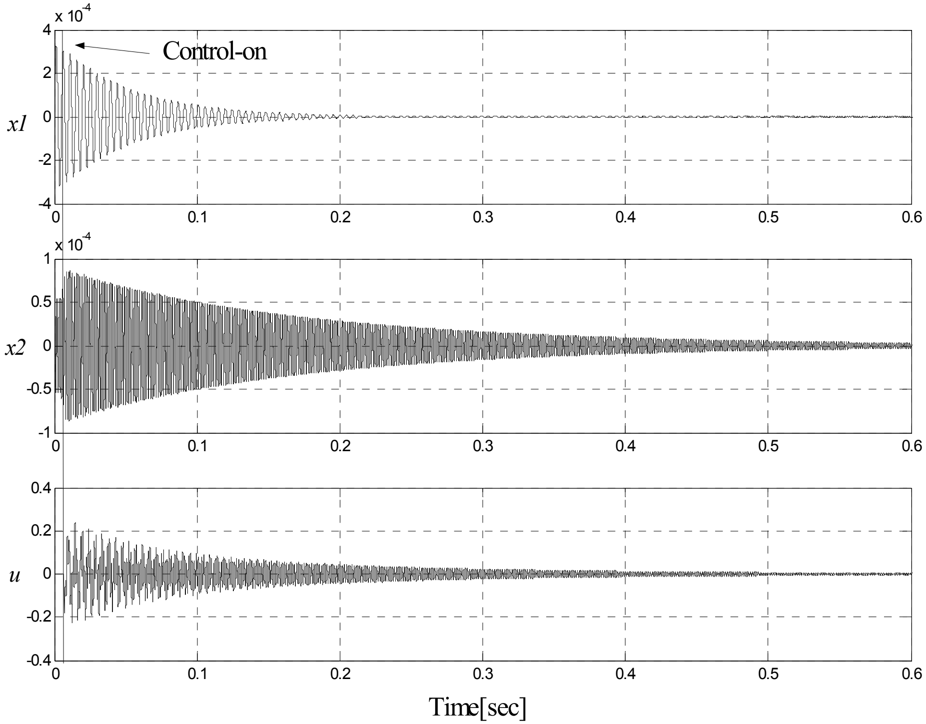
$$\begin{cases} K\varphi_{v0}\cos(\phi(\omega_1)) > 0, \\ K\varphi_{v0}\cos(\phi(\omega_2)) > 0, \end{cases}$$

- Quenching domain = validity domain of the K-B method.

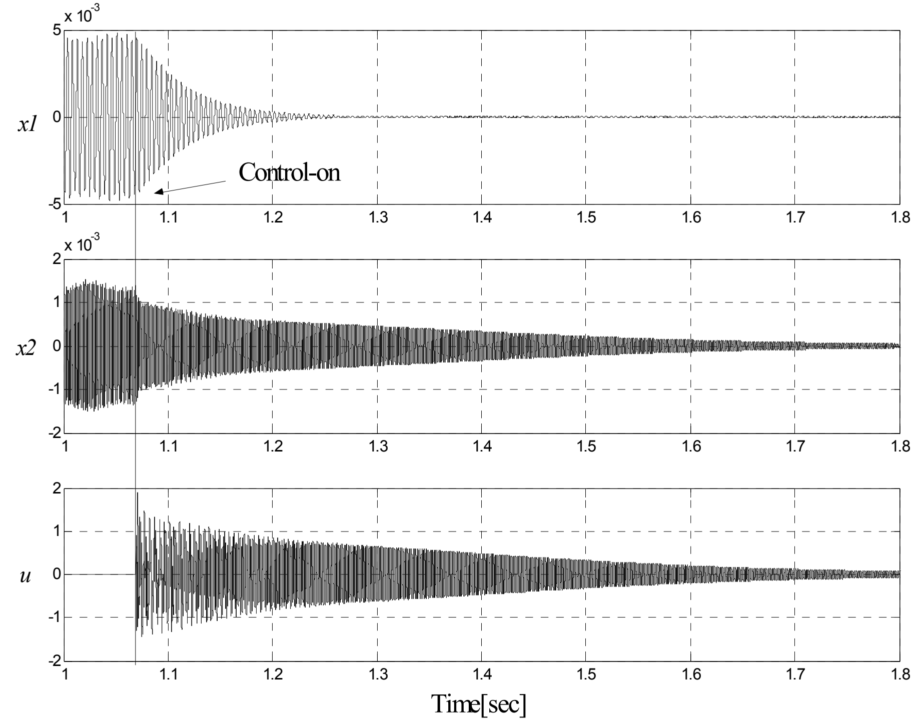
Quenching oscillations in combustion instability (4)

Simulation tests

Control applied at appearance of oscillations



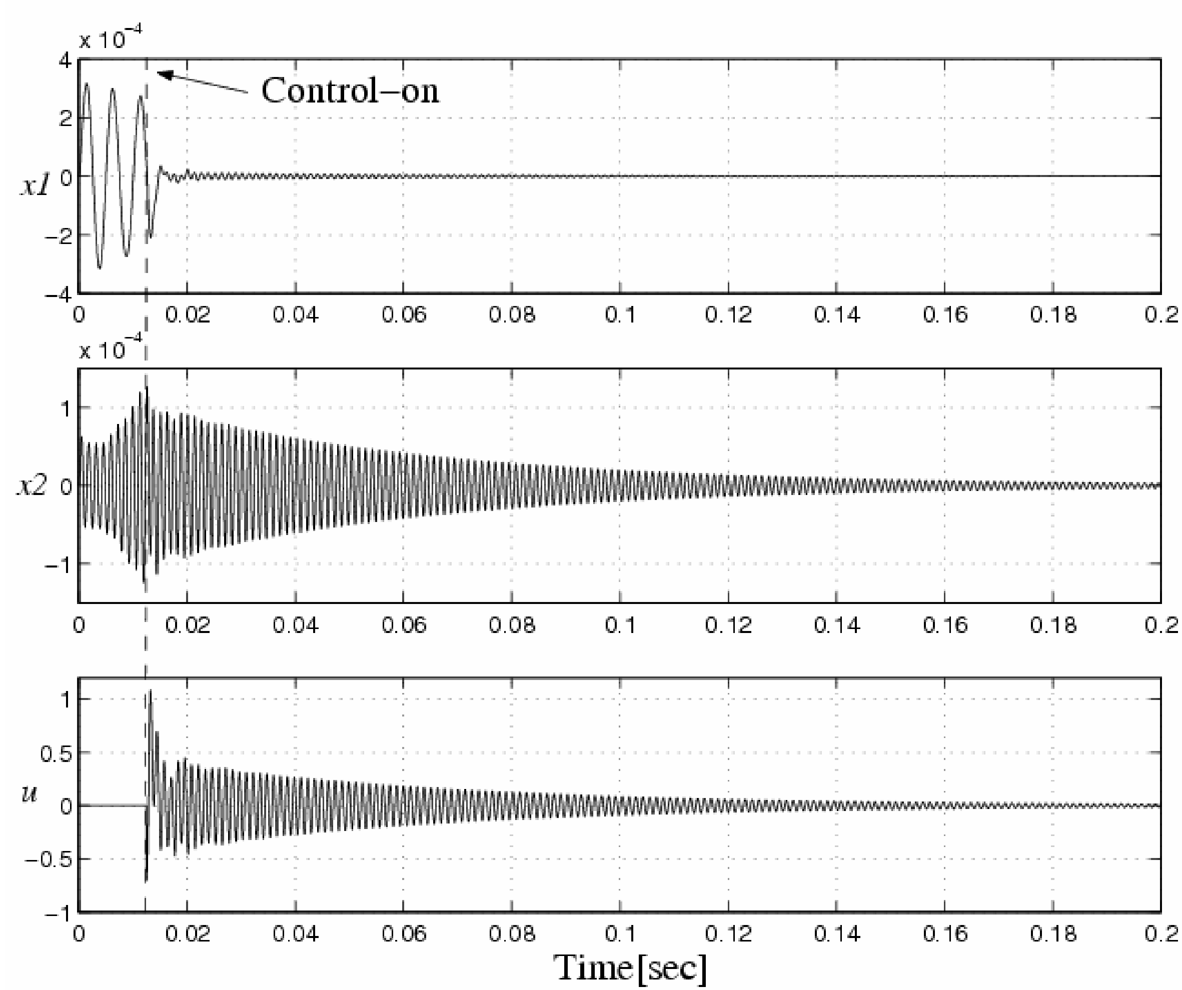
Oscillations already in steady operation



Perfect knowledge of model parameters

Robustness of oscillations quenching

5% error on model parameters



Gain K has to be augmented when parameter uncertainty goes up

I.D. Landau, F. Bouziani, R.R. Bitmead “Modelling and control of combustion instabilities”

Conclusion

• An analytically tractable model for combustion instabilities is proposed

• The model describes various phenomena observed in practice and captures the coexistence of two non-harmonic modes which occurs in real systems.

• Krylov-Bogoliubov method is proposed as a main tool for model analysis and control design.

• Control methodologies for quenching oscillations in combustion instabilities have been proposed.

• A nonlinear feedback have been considered and conditions for quenching the oscillations have been established.

• The simulations tests show that quenching was successful even in presence of errors in estimated model parameters.

References

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