Major advances in control

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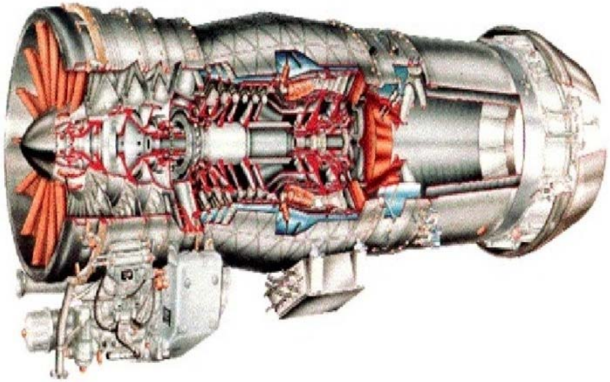


OUTLINE

Objective : Give an overview of some works developed in LAAS-CNRS in Control design.

The presentation is non exhaustive and presents only few works with few technical details.

I. Main works in control design until eighties
II. Robust control
III. Numerical tools
IV. Control with saturations
V. An example
VI. Perspectives

I. Main works in control design until eighties

LAAS was created in 1968 (Laboratoire d'Automatique et de ses Applications Spatiales – Laboratory of Automatic control and its Spatial Applications)

- ❑ Geographic environment : CERT-ONERA, CNES, SUPAERO, ENSICA, ENAC, INSA, INP, UPS…

- ❑ Major advances in control during sixties: **Kalman, Zadeh, Zames, Bellman**… EAST SCHOOL (**Lyapunov, Popov, Yakubovitch, Pontryaguine**…)

- ❑ Applications of these new theories in different domains : Spatial, Aeronautics, other industrial applications.

- ❑ Analog simulations and beginning of numeric simulations

During seventies and middle eighties

- Some works based on the new theories and methods : Stochastic processes, filtering, optimal control, Lyapunov stability, Nonlinear systems and others.

- Theories relatively well understood with theoritical guarantees, mathematically elegant and powerful.

- The main problems being for realistic systems :

- Modelling
- Numerical problems and computations in particular for

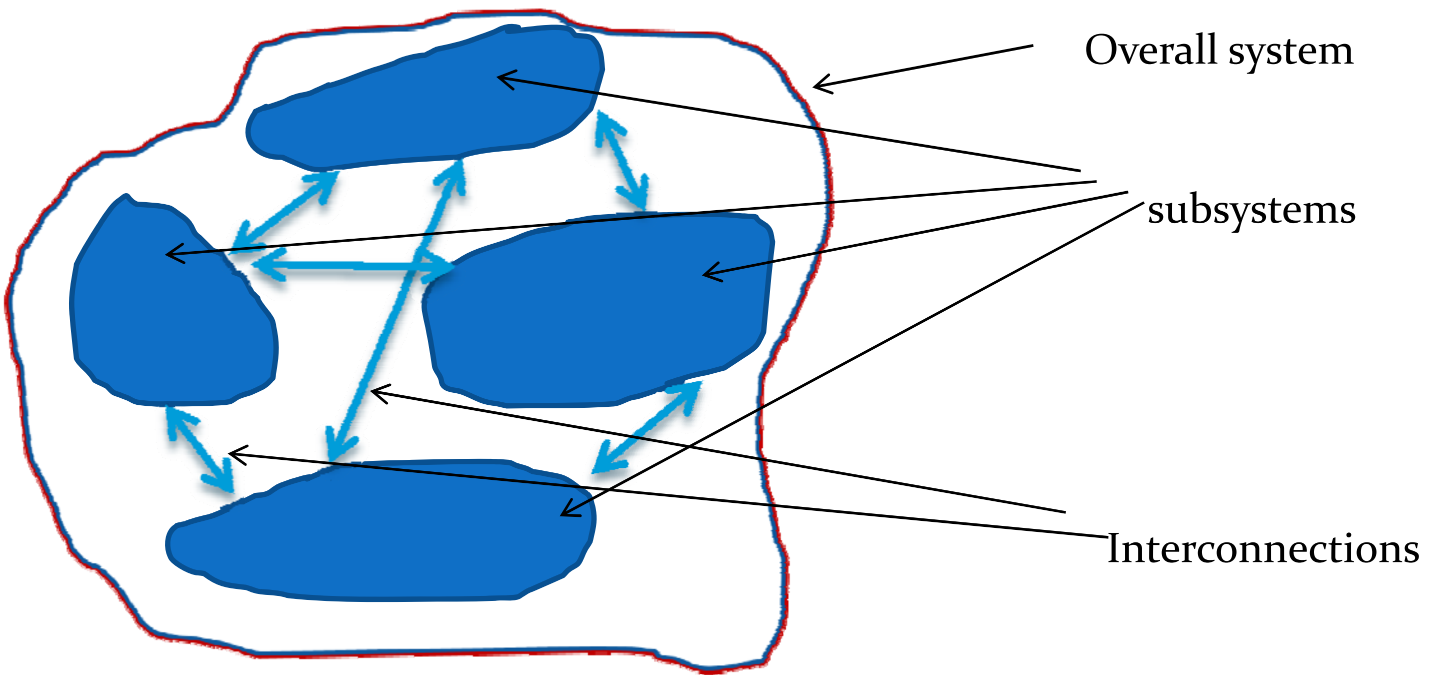
Analysis and control of complex and large scale systems

To solve these problems :

- ❑ Solve a collection of smaller problems

- ❑ Physical decomposition (structural constraints, Physical subsystems with low interconnections)

- ❑ Mathematical decomposition (Structural analysis, Quasi independent mathematical subproblems)



Until the middle of eighties, LAAS highly contributed to the analysis and control of complex and large scale systems with several applications in the domains of:

- Aerospace : *optimal control, dynamic programming, …*
- Networks : *stochastic control, routing, traffic control, power systems, communications…*
- Process control: *biotechnological processes, adaptive control, Nonlinear control …*

These problems induced some fundamental and methodological researches but also at the beginning of the eighties some works concerning numerical and algorithmic methods :

- *Decomposition and partitionned numerical methods*
- *Parallel algorithms*
- *Distributed algorithms*

At the end of the seventies and beginning of the eighties, personnal computers (PC) and workstations were developed and in the context of analysis and control, the first versions of MATLAB appeared.

- It was the beginning of a new period in which computing power and the development of numerical methods increased significantly. Several methods developed in the past and not easily applicable in realistic applications have been revisited .

- At the same time, some doubts affects the control community. During two decades, some important theoritical contributions around the Kalman's work and others lead to elegant methods with strong theoritical justifications , but in some practical cases, they fail and the observed results are not in accordance with theory.

- To simply illustrate the problem, we consider the well known observer-based control :

II. Robust control

Consider the following system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

With the control law

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \\ u = K\hat{x} + Hy_c \end{cases}$$

In closed loop, defining $e = x - \hat{x}$, we have

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A+BK & -BK \\ 0 & A-LC \end{pmatrix} \begin{pmatrix} x \\ e \end{pmatrix} + \begin{pmatrix} BH \\ 0 \end{pmatrix} y_c \\ y = \begin{pmatrix} C & 0 \end{pmatrix} \begin{pmatrix} x \\ e \end{pmatrix} \end{cases}$$

- Separation principle
- dynamic of error inobservable
- Closed loop system

$$C(sI - A - BK)^{-1} BH$$

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MODEL

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MODEL

In closed loop, defining $e = x - \hat{x}$, we have

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A + \Delta A + (B + \Delta B)K & -(B + \Delta B)K \\ \Delta A - L\Delta C + \Delta BK & A - LC - \Delta BK \end{pmatrix} \begin{pmatrix} x \\ e \end{pmatrix} + \begin{pmatrix} (B + \Delta B)H \\ \Delta BH \end{pmatrix} y_c \\ y = \begin{pmatrix} (C + \Delta C) & 0 \end{pmatrix} \begin{pmatrix} x \\ e \end{pmatrix} \end{cases}$$

Loop Transfer Recovery

(Doyle and Stein 1979, 1981)

- Closed loop system

$$C(sI - A - BK)^{-1}BH$$

The idea is to modelize uncertainties $\Delta A, \Delta B, \Delta C$ and take them into account in the control design synthesis $\longrightarrow$ **ROBUST CONTROL**

In LAAS, the quadratic stabilizability concept (Lyapunov approach) (**Barmish,1985**) was first studied (around 1986).

The main success of the developed approach was to associate to a control design problem an optimization problem (convex in numerous case). This was a central idea.

To illustrate this point, we present a seminal result obtained in LAAS in 1987.

$$\dot{x} = A x + B u$$

$$A = \sum_{i=1}^{N_A} \lambda_i A_i, \quad \sum_{i=1}^{N_A} \lambda_i = 1, \quad 0 \leq \lambda_i \leq 1$$

$$B = \sum_{i=1}^{N_B} \beta_i B_i, \quad \sum_{i=1}^{N_B} \beta_i = 1, \quad 0 \leq \beta_i \leq 1$$

If there exist matrices S=S'>o and R satisfying

$$A_i S + S A_i^T + B_j R + R^T B_j < 0 \quad i = 1,\ldots,N_A, \quad j = 1,\ldots,N_B$$

Then the controlled system by a state feedback $u = R S^{-1} x$ *is robustly stable*

- The set of matrices *S and R* defined by inequalities is convex. In the certain case, a complete parametrization of state feedback stabilizing gains is obtained.

- At beginning, the faisibility problem was tested by cutting plane technique. A sequence of linear optimization problems of increasing size are solved iteratively and the scheme theoritically converges to a solution when it exists.

- More interesting numerical approaches exist since the beginning of nineties) based on interior point algorithms (**Nemirovski, Gahinet 1995**) which are very efficient for solving Linear Matrix Inequalities (LMI)

$$F(z) = F_0 + \sum_{i=1}^{N} F_i z_i > 0, \quad F_i = F_i^T$$

A classical example is the following (**Lyapunov 1892**):

The system decribed by $\dot{x} = Ax$ *is asymptotically stable if and only if there exists* $P = P^T > 0$ *such that :*

$$A^T P + PA < 0$$

$$P = z_1 \underbrace{\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{P_1} + z_2 \underbrace{\begin{pmatrix} 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{P_2} + \cdots + z_{\underbrace{n(n+1)/2}_{N}} \underbrace{\begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{P_N}$$

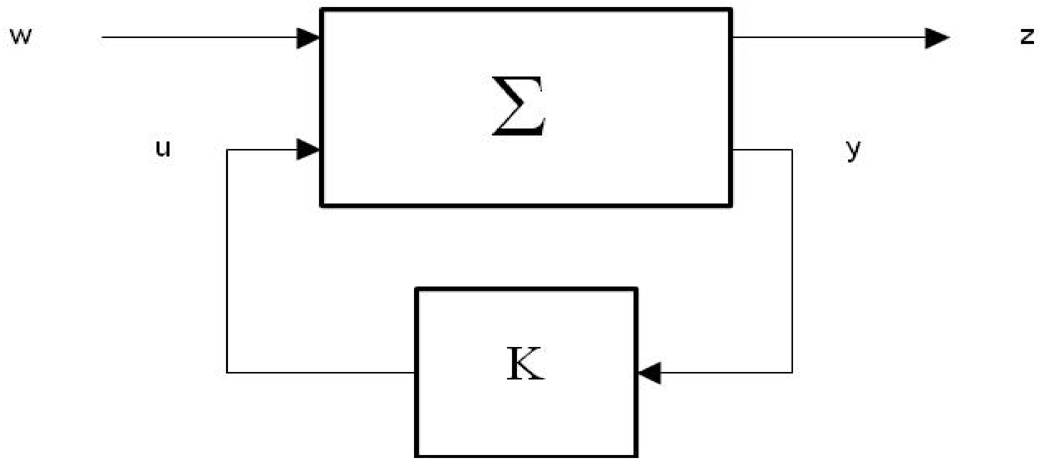
$$-A^T P - PA = \sum_{i=1}^{n(n+1)/2} z_i (-A^T P_i - P_i A_i) = F_0 + \sum_{i=1}^{N} F_i z_i > 0,$$

$$F_i = F_i^T = -A^T P_i - P_i A, \quad F_0 = 0, \quad N = n(n+1)/2$$

The concept of LMI was introduced by **Willems, 1972.**

❑ Another important *idea* was to obtain a generic formulation of a control design problem through a standard problem formulation. This formulation translates the control design problem into an optimization problem in functional spaces. The concept of norms of signals and operators are central.

This approach was in particular suggested by **Zames , 1981** and leads to the standard problem



- z: controlled output
- y: measured output
- u: control
- w: error signal, perturbations…
- $\Sigma \in \boldsymbol{\Sigma}$

$$\min_{u=K(y)} \left\| T_{zw}(s) \right\|_{2 \text{ or } \infty}$$

The problem was solved by **Doyle et al in 1989**. Riccati equations play a crucial role (*top 10 most influential papers in mathematics 1981-1993*). LMI formulations were derived by **Gahinet et al. in 1994**.

❑ Numerous control problems can be dealt with in this way and translated into LMI optimization problems (LQ, LQG, H2, Hinfinity and their robust counterparts : guaranteed cost controls).

❑ The contributions of LAAS are numerous and recognized. In particular, some improvements relaxing the necessity of a fixed Lyapunov function (quadratic stability) were proposed in 1999 and 2000. These new conditions are among the best published in the literature and were followed by numerous publications in the context of control design and filtering

❑ Some extensions to a class of linear parameter varying systems were also considered. They are written as (**Gahinet and Apkarian 1995**)

$$\dot{x} = \left(\sum_{i=1}^{N} \lambda_i(\theta) A_i \right) x + \left(\sum_{j=1}^{M} \beta_j(\theta) B_j \right) u$$

$\theta \in \Theta$ measured parameters

- Some extensions to a class of non linear systems were also considered. They are written as:

$$\dot{x} = \left(\sum_{i=1}^{N} \lambda_i(x) A_i \right) x + \left(\sum_{j=1}^{M} \beta_j(x) B_j \right) u$$

With some additional properties on $\lambda_i(.)$ and $\beta_j(.)$, this model can also represent a Takagi-Sugeno model and is related to fuzzy control.

Some works developed in the context of robust control can be used to qualify fuzzy controls strategy (stability, performances).

All these works are supported by reliable LMI solvers (Several toolboxes exist and in LAAS some interfaces were developed for solving analysis or control design problems). This is a major advance. But all the interesting practical problems cannot be formulated through LMI inequalities.

III. Numerical tools

The simple static output feedback stabilization problem cannot be translated into a convex feasibility problem.

Consider the following system

$$\begin{cases} \dot{x} = A\,x + B\,u \\ y = C\,x \end{cases}$$

The problem is to characterize the family of the static output stabilizing controls, that is

$$u = Ly$$

*System is stabilizable by static output feedback if and only if there exist a positive definite symmetric matrix P and a matrix L satisfying (**Lyapunov**):*

$$(A + BLC)P + P(A + BLC)^T < 0$$

❑ Up to now an equivalent convex formulation of this problem does not exist. Only sufficient condition can be obtained. Many of them were developped in LAAS.

❑ The apparent simplicity of the problem formulation masks a real numerical complexity (**Blondel, Tsitsiklis, 1995**).

❑ Some important control design problems can be formulated through an equivalent static output feedback control design or at least possess the characteristics of a static output feedback design. We can cite for example:

- Decentralized control
- Reduced order control
- Multiobjective control
- and others in which structural constraints play a key role.

Recently a lot of work has been done in that direction in LAAS.

- Several numerical tools were developed for solving control problems :

  - **RoMuLoc**: Matlab toolbox for robust multiobjective analysis and synthesis using YALMIP parser and SDP solvers SeDuMi, CSDP,DSDP, SDPT3...
(http://www.laas.fr/OLOCEP/romuloc/)

  - **Polynomial toolbox** : Commercial software for manipulating polynomials and solve control and signal processing problems with Matlab. *(http://www.polyx.cz)*

- More recently, at LAAS a new method based on the general moment theory has been developed.

The general moment problem can be formulated as follows:

$$\max_{\mu \in M(S)} \left\{ \int f_0 \, d\mu : \int f_j \, d\mu = b_j, \; j = 1, \ldots, m \right\}$$

- If $S \subset R^n$ is a semi-algebraic set and f_j's are piecewize polynomials, using recent results obtained in the field of algebraic geometry , it is possible to solve the moment problem using semidefinite programming (SDP) (a convex optimization method).

- In fact, a hierarchy of relaxations of initial problem has to be solved. Each relaxation is a semidefinite programming problem (convex optimisation).

- The sequence of optimal values converges monotonically to the solution of the initial problem. The efficiency of the approach closely depends of the efficiency of SDP solvers

Among the main applications, we can cite:

- ❑ Global optimization (continue and discrete)
- ❑ Robust and optimal control (analysis and synthesis)
- ❑ Performance evaluation
- ❑ Computer vision
- ❑ Quantum information…

A tool for solving convex linear matrix inequality relaxations of the global optimization problem of minimizing a multivariable polynomial function subject to polynomial inequality, equality or integer constraints was developed :

GloptiPoly : *http://www.laas.fr/~henrion/software/gloptipoly*

IV. Control with saturations

- In practice, some nonlinearities have to be taken into account when designing the control law. Some of them cannot be treated by simple extensions , this is the case of saturations due to the limits of actuators or sensors in position or in rate.

- Saturated control was an old subject and LAAS was one of the first laboratory which investigated this problem. During the last decade, this problem concentrated the interest of a lot of people (**Saberi, Lin, Teel, Posthlewaite, Turner, Gladdfelder, Sontag, Jabbari, Stoorvogel**…)

- The effects of a saturation in position is well understood in practice and was solved **heuristically** in industry by anti-windup techniques. The case of rate saturations is more involved and their effects not very known. Here two examples : (Pilot Induced Oscillations, PIO, Garteur projects)

Consider the system described by:

$$\dot{x} = Ax + Bsat(u)$$

where sat(u) is defined as :

$$sat(u_{(i)}) = \begin{cases} u_{0(i)} & \text{if } u_{(i)} > u_{0(i)} \\ u_{(i)} & \text{if } |u_{(i)}| \leq u_{0(i)} \\ -u_{0(i)} & \text{if } u_{(i)} < -u_{0(i)} \end{cases}$$

The closed-loop system is nonlinear. If the system does not possess some open-loop stability properties, the system is only locally stabilizable and it is important in practice to obtain a good approximation of the stability domain

$\longrightarrow$ Use of Lyapunov stability theory and its extensions (Lasalle invariance principle) and tools such that : Finsler lemma, S-procedure, Schur complement…

Idea : Replace the saturations by an approximation only valid around the equilibrium point and try to maximize the domain of validity of the approximation which also contains an approximation of the domain of stability

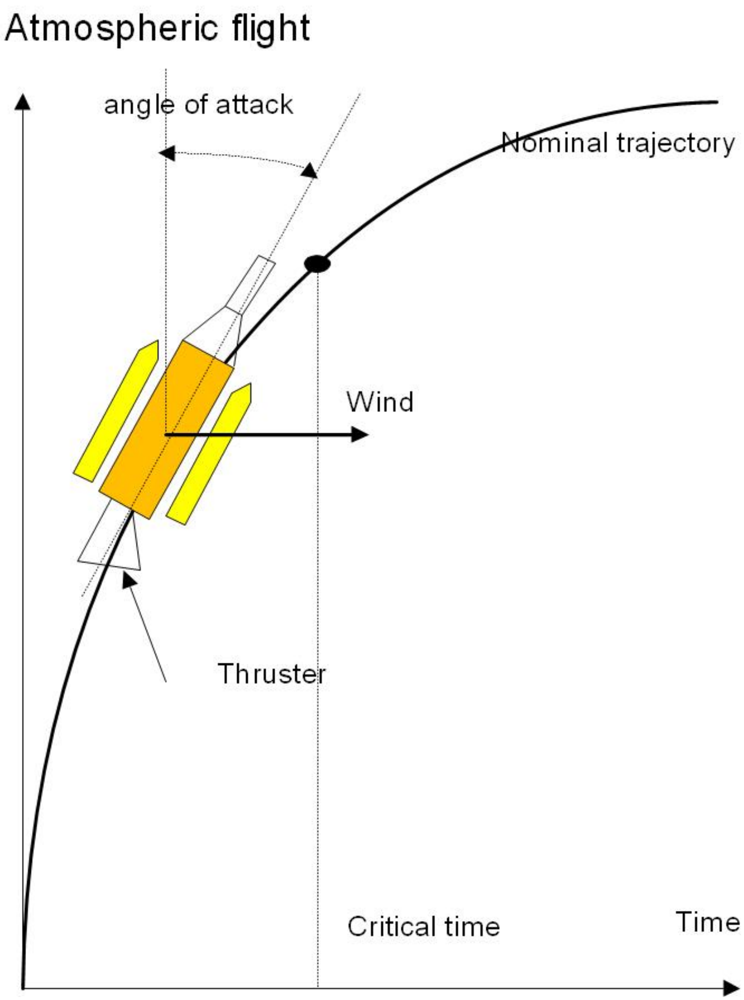
Two approaches :

- $$sat(u) = \sum_{j=1}^{2^m} \lambda_j(u) D_j u, \quad \sum_{j=1}^{2^m} \lambda_j(u) = 1$$

Polytopic approach leads to Bilinear Matrix Inequalities (BMI) conditions and fixing the domain of validity of the approximation leads to LMIs ⟶ Numerically tractable

- Interpret sat(u) as a sector non-linearity and with **an extension of the sector nonlinearity condition developed in LAAS**, we obtain LMI conditions. It is not necessary to a priori fix the domain of validity of the approximation. It is also possible with this approach to easily deal with nested saturations sat(sat(.))

V. Control of a space launch vehicle



- open-loop unstable aerodynamical system

- Large parametric uncertainties

- Constraints on
  - *thruster angle of deflection,*
  - *derivative of angle of deflection,*
  - *angle of attack*

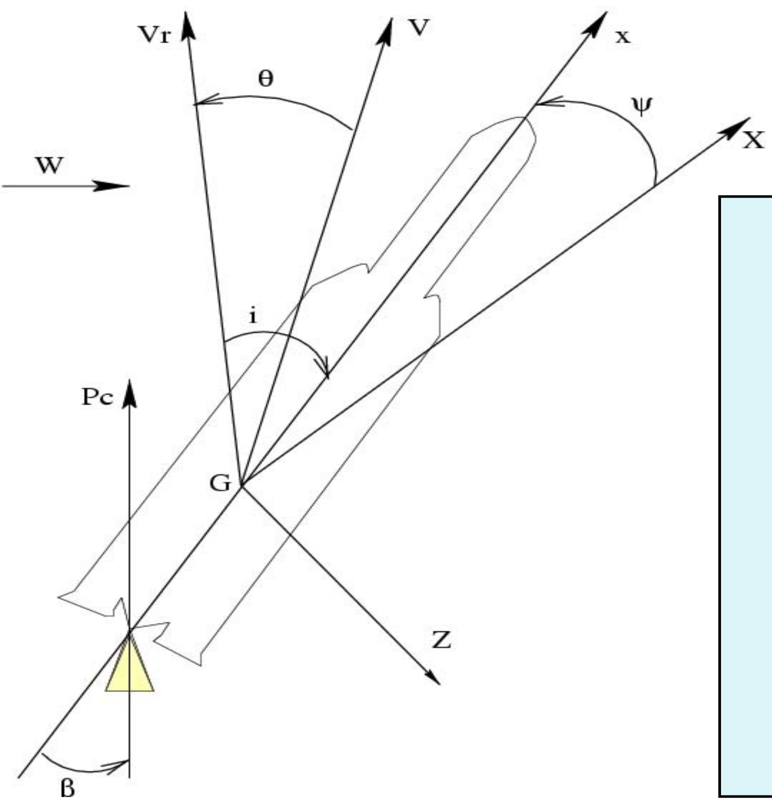
- Perturbations: wind, vibrations…

Rigid Model

$$x = \begin{bmatrix} e_1 \\ \dot{e}_1 \\ \Psi \\ \dot{\Psi} \\ \dot{z} \end{bmatrix}$$

$$u = \beta \ , |\beta| \leq 6°$$

$$z = \begin{bmatrix} i \\ \dot{z} \end{bmatrix} \quad y = \begin{bmatrix} \Psi \\ \dot{\Psi} \end{bmatrix} \quad |i| \leq i_0$$



$$\dot{x} = A(\Delta)x + B_u(\Delta)u + B_w(\Delta)w$$

$$z = C_1(\Delta)x + D_1(\Delta)w$$

$$y = C_2(\Delta)x$$

Δ *: Uncertain parameters and time-varying parameters*

A flexible mode model was also considered (order=15)

Robust control synthesis

Specifications translated into norm constraints

$$\min_{K \in \mathcal{K}} \alpha_i \gamma_{vent\_i} + \alpha_{conso} \gamma_{conso} \qquad \alpha_i, \alpha_{conso} > 0$$

$$\left\| \Sigma_{mod} * K \right\|_{\infty}^{2} \leq \gamma_{mod}$$

$$\left\| \Sigma_{vent\_i} * K \right\|_{2}^{2} \leq \gamma_{vent\_i}$$

$$\left\| \Sigma_{conso} * K \right\|_{2}^{2} \leq \gamma_{conso}$$

Translation into linear matrix inequalities(LMI)

$$\min \quad \alpha_i \gamma_{vent\_i} + \alpha_{conso} \gamma_{conso} \qquad \alpha_i, \alpha_{conso} > 0$$

$$\Psi_{\infty}(P_m, J_m, H_m, X, Y, S, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \gamma_{mod}) < 0$$

$$\Psi_{i2p}(P_i, J_i, H_i, X, Y, S, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \gamma_{vent\_i}) < 0$$

$$\Psi_{co}(P_{co}, J_{co}, H_{co}, T_{co}, X, Y, S, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \gamma_{conso}) < 0$$

Robust control synthesis

Specifications translated into norm constraints

$$\min \; \alpha_i \gamma_{vent\_i} + \alpha_{conso} \gamma_{conso}$$

Controller

$$\begin{cases} \dot{\eta}_c = A_c \eta_c + B_c y \\ u = C_c \eta_c + D_c y \end{cases}$$

$$V_1' U_1 = S - YX$$

$$D_c = \overline{D}$$

$$C_c = (\overline{C} - \overline{D} C X) U_1^{-1}$$

$$B_c = V_1^{-T} (\overline{B} - Y B \overline{D}) U_1^{-1}$$

$$A_c = V_1^{-T} \left[\overline{A} - Y(A + B \overline{D} C) X - V_1' B_c C X - Y B C_c U_1 \right] U_1^{-1}$$

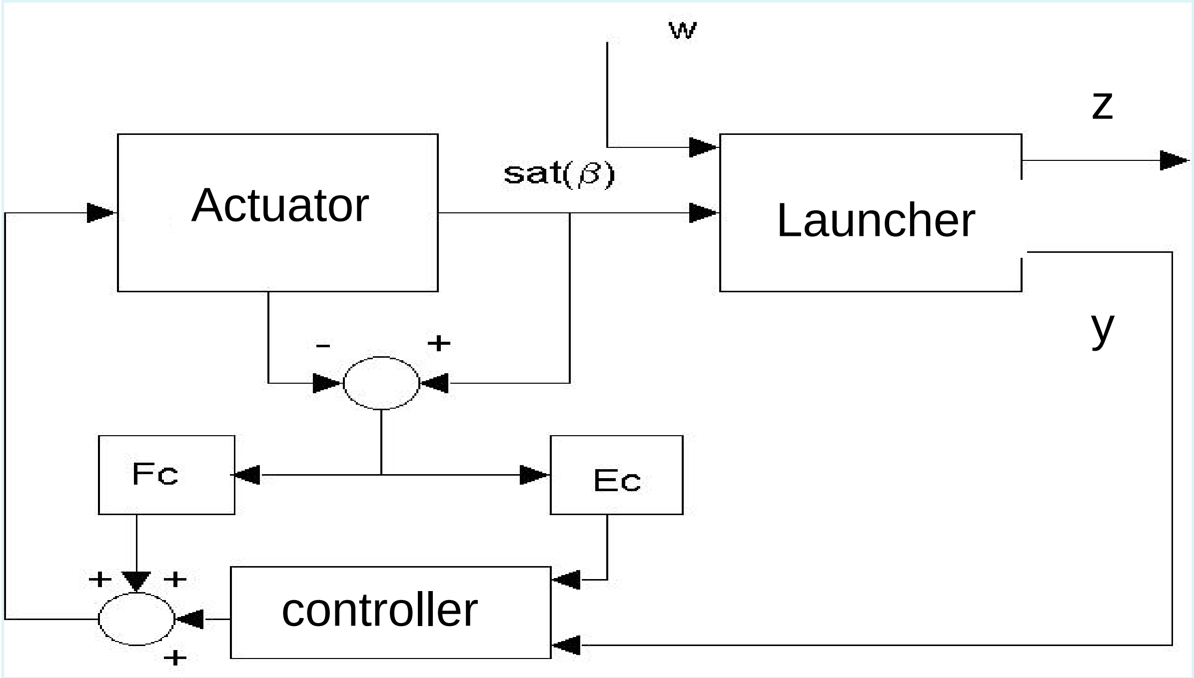
Tr

$$\Psi_\infty (P_m, J_m, H_m, X, Y, S, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \gamma_{mod}) < 0$$

$$\Psi_{i2p} (P_i, J_i, H_i, X, Y, S, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \gamma_{vent\_i}) < 0$$

$$\Psi_{co} (P_{co}, J_{co}, H_{co}, T_{co}, X, Y, S, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \gamma_{conso}) < 0$$

Anti-windup approach



The control law is modified and the added terms are only active when the control saturates :

$$\dot{\eta} = A_c\eta + B_c y + E_c[sat(\beta) - \beta]$$
$$y_c = C_c\eta + D_c y + F_c[sat(\beta) - \beta]$$

The region of stability is maximized while respecting the constraints on incidence.

VI. Perspectives

These last years, some important progresses have been done in several domains:

- Actuators and sensors : at low costs, with a reduced size,
- Computers and microcontrollers : at low costs, powerful and flexible,
- Networks : of sensors, actuators and computers allowing a large scale instrumentation of the systems

These progresses have several impacts in a lot of application domains and open new domains of applications (biology, ecology…). Some ideas unrealistic some years ago are now possible, in particular:

- Real-time systems taking into account a high degree of security and complexity
- Embedded systems with a certain level of autonomy , reconfigurable and intelligent.

The control of such systems is more complex and must take into account important constraints. Some of them are listed below

- ❑ The structure of control laws (complexity, flexibility),
- ❑ Limited information (due for example to limits induced by coding or quantification , saturations, delays,…)
- ❑ Real time constraints (related to the structure of control laws, priorities, sampling)
- ❑ Uncertainties, nonlinearities, failures (sensors, actuators, system)
- ❑ Problems of energy management
- ❑ Interplay between hardware and software (hybrid models)
- ❑ …

- Non linear and hybrid control
  - take into account nonlinearities in general and the ones due to sensors and actuators in particular
  - Control and estimation
  - Control reconfiguration (reset control, hybrid models…)
- Infinite dimensional systems
  - Systems with an important degrees of freedom (due for example to distributed sensors or actuators)
  - Find adapted strategies with reliable numerical methods
- Reduced complexity control
  - take into account structural contraints in the control laws (reduced order, decentralized…)
  - Uncertainties (a wide variety of uncertainty descriptions)
  - Multiobjective control (try to translate specifications into adapted measures)

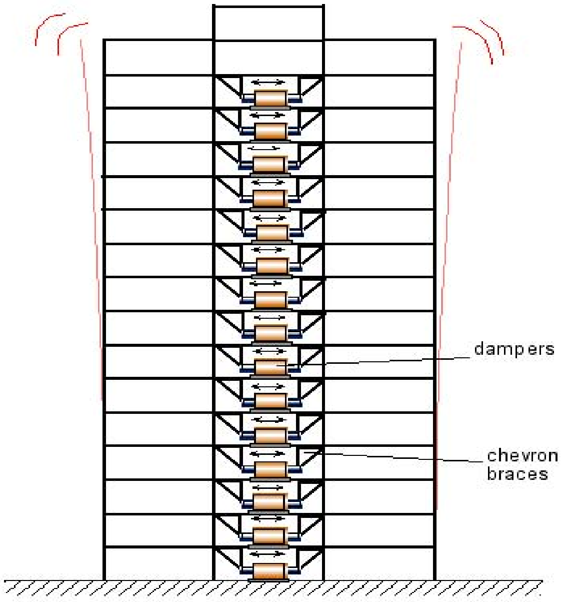
- Networked controlled systems
    - Modeling (delays, network protocols…)
    - Control design with guaranteed properties (performance, stability)

- Optimization
    - Global optimization
    - Tools for analysis and control synthesis (state space, polynomial descriptions…)

Some Applications

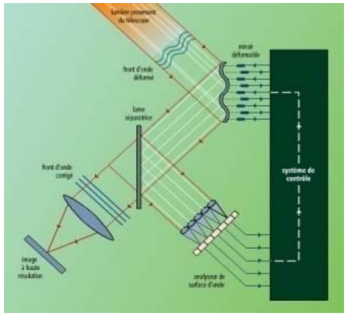
Flexible structures with piezo electric sensors and actuators

Aeronautic



Vibration suppressions in robots

Spatial



Adaptive optic

THANK YOU

FOR

YOUR ATTENTION